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A Crosscorrelation Method for Measuring the Impulse Response of Reactor Systems*

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A crosscorrelation method for the study of the small signal response of nuclear reactor systems is presented. The technique and equipment necessary for the implementation of the method are discussed. Results from experiments performed on the Godiva and Kiwi-A3 reactors are reported.

Experiments have shown that the method and the technique give very satisfactory results under adverse experimental conditions.

I. INTRODUCTION

The small signal dynamic response of reactor systems is normally measured either by exciting the reactor with sinusoidal or step changes of reactivity or by autocorrelating the reactor power fluctuations. These techniques each have inherent restrictions which limit their use.

Sinusoidal or oscillation tests require a long operating time because each frequency component of the response spectrum is excited separately. In addition, the response must be appreciably larger than the inherent reactor noise to obtain accurate results unless input–output crosscorrelation techniques are employed.

Step response experiments do not require a long time but such an experiment can not always be performed. The minimum step amplitude is determined by the accuracy required in the presence of the inherent reactor noise; the maximum step amplitude is determined by safety considerations or the requirement that the response be contained in a linear range of the system. For many reactor applications there is no step amplitude which satisfies these requirements.

Reactor output power autocorrelation methods do not introduce system disturbances but do require knowledge of the statistical processes in the reactor system. This information is usually not well known.

The impulse response of a linear system can also be determined by exciting the system with a noise-like input and crosscorrelating the input with the system output (*1*). This method requires relatively little time and gives good results for low-level crosscorrelation signal inputs whose power is comparable to the noise power in the system. Thus, measurements can be made without disturbing the system operation significantly, and can even be made while other inputs to the system are changing.

The purposes of this paper are to describe the impulse response crosscorrelation method as it has been applied to nuclear reactor systems and to report on results from two reactor experiments. The following section is devoted to a mathematical description of the crosscorrelation method and the requirements on the input imposed by the practical necessity of performing the experiment over a limited interval of time. The third section gives a description of the equipment used in the experiments. The

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fourth section reports results from experiments on the Godiva and Kiwi-A3 reactors. The paper concludes with a general discussion of the advantages of the method and its possible use as a reactor stability monitor.

II. MATHEMATICAL DESCRIPTION OF THE METHOD

Consider a linear system characterized by an impulse response or weighting function $h(t)$. For any input $a(t)$, the corresponding system output, $b(t)$, is given by the convolution integral

$$b(t) = \int_0^\infty h(\lambda)a(t-\lambda)\,d\lambda \tag{1}$$

The crosscorrelation function (*2*), $\phi_{ab}(\tau)$, between the input and the output is defined as:

$$\phi_{ab}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} a(t)b(t+\tau)\,dt \tag{2}$$

Substituting Eq. (1) for $b(t)$ and reversing the order of integration, one obtains

$$\phi_{ab}(\tau) = \int_0^\infty d\lambda h(\lambda) \lim_{T\to\infty}\frac{1}{2T} \cdot \int_{-T}^{T} dt\cdot a(t)a(t+\tau-\lambda) \tag{3}$$

The integral with respect to t in Eq. (3) is defined as the autocorrelation function of the input $a(t)$.

$$\phi_{aa}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} a(t)a(t+\tau)\,dt \tag{4}$$

Thus, Eq. (3) reduces to

$$\phi_{ab}(\tau) = \int_0^\infty h(\lambda)\phi_{aa}(\tau-\lambda)\,d\lambda \tag{5}$$

If the autocorrelation function of the input is a delta function,

$$\phi_{aa}(\tau) = \delta(\tau) \tag{6}$$

then the crosscorrelation function of the input and the output is the impulse response of the system.

$$\phi_{ab}(\tau) = h(\tau) \tag{7}$$

Thus, if an appropriate input signal is selected, the linear system input–output crosscorrelation function is identical to the system impulse response. The work reported in this paper is based on the validity of the identity as expressed in Eq. (7).

The system transfer function can be found by taking the Laplace Transform of Eq. (7)

$$\bar{\phi}_{ab}(s) = \bar{H}(s) \tag{8}$$

It should be noted that Eq. (7) holds only for a time-invariant system. On the other hand, Eq. (7) does hold even when there are other command or noise signal inputs to the system so long as these signals do not correlate with $a(t)$.

The relationships which must be implemented to apply the crosscorrelation method to the study of nuclear reactor dynamics are given in Eqs. (2) and (6); and the result of the crosscorrelation is given in Eq. (7). Practical considerations require use of approximations to both Eq. (2) and Eq. (6).

First, the correlation time must be finite; and it is necessary to use a modified form of Eq. (2), as shown below.

$$\phi'_{ab}(\tau) = \frac{1}{T}\int_0^T b(t)a(t-\tau)\,dt \tag{9}$$

We will consider integration times, T, of the order of the settling time of the system, which is the time at which the impulse response reaches a negligible value.

Second, the input signal must have finite bandwidth but still retain an autocorrelation function which is approximately a delta function. One such signal can be defined as follows:

(a) It is binary and assumes values of plus or minus one with equal probability.

(b) It may (but need not necessarily) change sign only every Δt seconds.

(c) It is periodic with period $T = N\Delta t$, $N \gg 1$.

A typical signal of this type, and its autocorrelation function, are shown in Fig. 1. The autocorrelation function does look somewhat like a delta function in every period T. However, it is also accompanied by side lobes because of the finite signal length. The side lobe amplitude probability distribution is the binary probability distribution, and for large N is nearly normal with $\sigma = 1/\sqrt{N}$. These side lobes introduce a sizable error in the crosscorrelation result unless N is very large.

It is possible to obtain binary signals or "chains" with a somewhat idealized autocorrelation function as shown in Fig. 2. In order to obtain a chain with this autocorrelation function one must have a sequence, $a_1, a_2, a_3, \cdots a_N$; $a_i = +1$ or -1, such that

$$\sum_{i=1}^{i=N} a_i\, a_{(i+j)\mathrm{mod}N} = \begin{cases} N, & j = 0 \\ -1, & j \neq 0 \end{cases} \tag{10}$$

This relation is known to be achievable in cases where N is prime and of the form $4k - 1$ (*3*). Chains possessing this property are known for $N = 251$ and $N = 1019$ (*4*). The desirability of this autocorrelation function, as compared to that which might be obtained by choosing a chain at random, is evident in the graphical description of Eq. (5), for an idealized chain, as shown in Fig. 3. In essence, the response function, $h(\lambda)$, is sampled by the function $\phi_{aa}(\tau - \lambda)$. Assuming that $h(\lambda)$ does not vary appreciably over the time interval Δt one has

$$\phi'_{ab}(\tau) \cong \Delta t \left[h(\tau) - \frac{1}{T} \int_0^T h(\lambda)\, d\lambda \right] \quad (11)$$

The right term in brackets above accounts for the fact that $\phi_{aa}(\tau - \lambda)$ has a constant value, $-1/N$, almost throughout the entire range T. This correction term can be evaluated by considering a slightly negative shift, $\tau = -\Delta t$ or $\tau = -2\Delta t$. The correction is constant and can be included in the results.

III. EQUIPMENT

The equipment necessary for the application of the crosscorrelation method to a nuclear reactor system consists of: (1) a means of supplying the special binary input, and (2) a crosscorrelator. Such

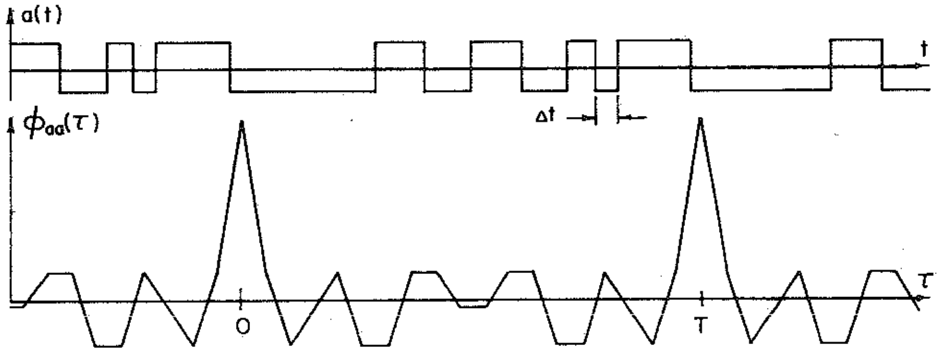

FIG. 1. Random periodic binary signal for $N = 19$ and the corresponding autocorrelation function. The autocorrelation function has the same periodicity as the signal.

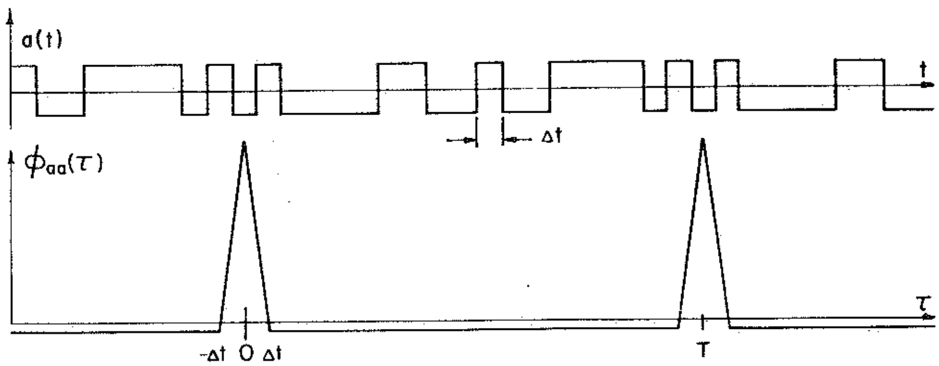

FIG. 2. Idealized periodic binary signal for $N = 19$ and the corresponding autocorrelation function

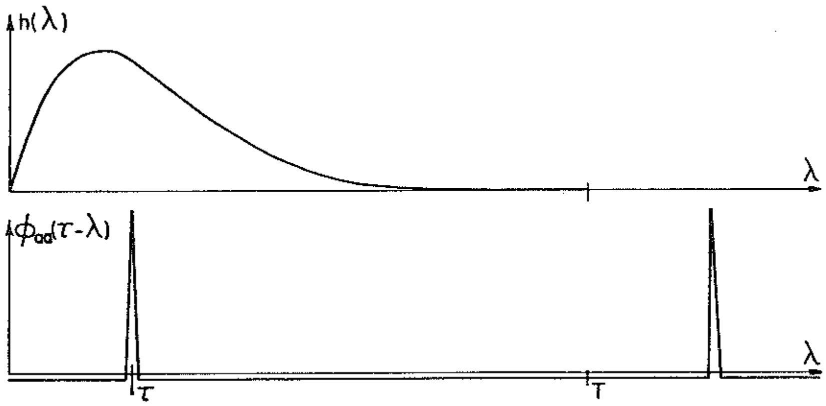

FIG. 3. Graphical description of Eq. (5) for an idealized input

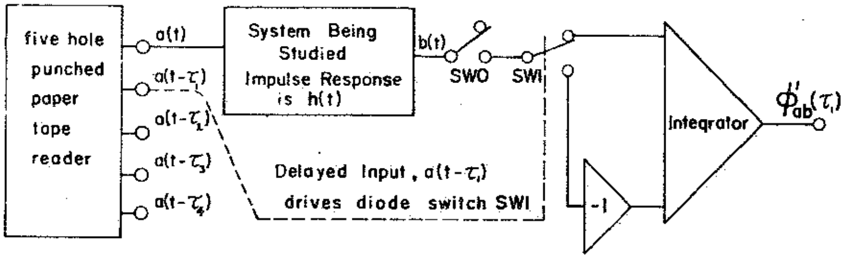

FIG. 4. Schematic diagram of the input signal generator and crosscorrelation function calculator. $\phi'_{ab}(\tau_i) \approx \Delta t \cdot h(\tau_i)$ when integration over one period, T, is completed.

equipment has been built. A schematic diagram is shown in Fig. 4.

The input and delayed inputs for four different values of the delay time, τ_i, are read from a five channel punched paper tape. The input signal excites the system under investigation. Each of the delayed input signals switches a diode gate to the plus one or minus one input of an integrator. The diode gate is fed by the output of the system. The switching implements the multiplication of the output, $b(t)$, and the delayed input, $a(t - \tau_i)$. At the end of the integration the value of each integrator output is the value of the crosscorrelation function, $\phi'_{ab}(\tau_i)$. This is approximately proportional to $h(\tau_i)$, the impulse response of the system evaluated at time τ_i. The integration is terminated by a relay operated switch, SWO, at the input of the integrator, which opens when a scaler started at $t = 0$ counts N, or a multiple of N, of the Δt spaced timing pulses.

The integration and sign reversal operations in the crosscorrelator are performed by standard high gain analog computer components with the proper feedback. The system under investigation must be in equilibrium with the input before integration is initiated but integration can be initiated anywhere within the periodic chain.

The diode switch, invertor, and integrator have been duplicated four times so that values of $\phi'_{ab}(\tau_i)$ for four different delays, τ_i, are computed simultaneously. The entire shape of the $\phi'_{ab}(\tau)$ or $h(\tau)$ curve can be determined by using different punched paper tapes with different groupings of delays, each computing four different points on the curve. Use of more channels in parallel would enable one to measure more points on the impulse response simultaneously.

Of course, it is not necessary to calculate the impulse response during the time the system is under test. One can store the input signal, the output signal, and the timing pulses on magnetic tape and crosscorrelate, using the same equipment described above, after the experiment is over. This procedure has also been used in the experiments that are reported in the next section.

The transfer function of the system can be derived from the impulse response by taking the Laplace Transform as shown in Eq. (8). Since the impulse response is given in terms of a number of data points, there is some ambiguity about the shape of the curve between the points. One can approximate this curve by a series of straight lines joining the data points. The resulting transfer function is given below.

$$\bar{H}(s) = \frac{1}{s^2} \sum_i \begin{pmatrix}\text{change in slose} \\ \text{at the } i\text{th point}\end{pmatrix} e^{-p\tau_i} + \frac{1}{s}\,(\text{value at } \tau = 0) \tag{12}$$

A 704 FORTRAN Program has been written to calculate $\bar{H}(s)$ using Eq. (12).

The crosscorrelation equipment that has been built to obtain the impulse response of a linear system has been described above. It is of course possible to implement the crosscorrelation method differently; and a discussion of some possible variations follows.

The input signal can be classified by two properties: (a) predetermined or stochastic, and (b) discrete level or continuous level. If the input signal is predetermined then the delayed signals are predetermined. This simplifies calculation of the crosscorrelation function because no pure time delay device is needed; however, it implies the need for a device to store the input signal and the delayed input signals. In addition, if the input is predetermined, a particular signal which has an autocorrelation function much like a delta function can be selected. A truly stochastic input can be generated by a radioactive source, for example, and must be stored only long enough to generate the appropriate delayed input.

A discrete level binary input offers a number of advantages. Such an input is easy to store and easy to delay. It is also easy to implement the multiplica-

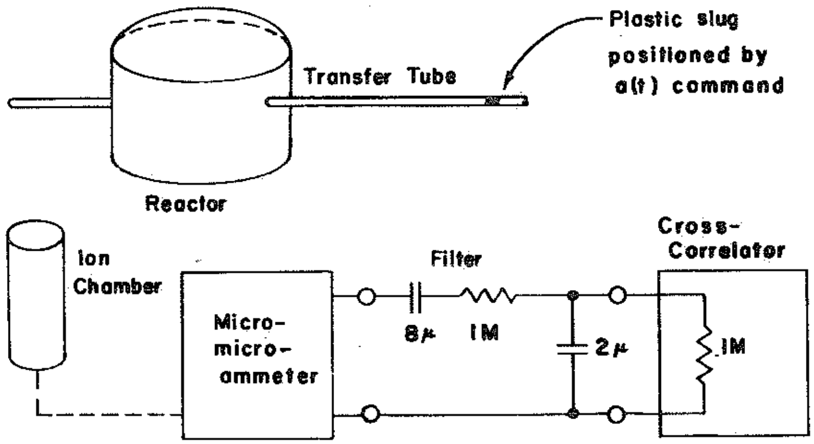

FIG. 5. Schematic diagram of the Godiva reactor experimental setup

tion of the binary input and the system output. On the other hand, a continuous level input signal would require the multiplication of two continuous signals in the computation of the crosscorrelation function; and this type of multiplication is relatively expensive to implement. It is also relatively difficult to implement the pure delay of a continuous signal with the accuracy that is required (*3*).

The crosscorrelation calculations can be carried out on a digital computer if a means is available to digitize the system input and system output.

IV. EXPERIMENTS

The equipment described in the previous section was first tested on analog computer representations of a variety of physical systems, including reactor simulations. It was found that the impulse responses can be measured quite accurately. Specifically, in the case of a second order system with a damping factor of 0.5 and a resonant frequency of 1 rad/sec, the data agree with the theoretical prediction of the impulse response within the following limits:

For input signal characteristics $N = 251$ and $\Delta t = 0.05$ sec:

(1) Impulse response:

(a) $$\frac{\text{maximum error}}{\text{maximum value of response}} = 2.5\%$$

(b) $$\frac{\text{r.m.s error}}{\text{maximum value of response}} = 1\%$$

(2) Transfer function derived from impulse response:

(a) Maximum decibel error = 0.5 db

(b) Maximum phase error

$= 2°$ up to 6 rad/sec

$= 10°$ up to 60 rad/sec

No noise simulator was available to establish the adverse effects of uncorrelated noise experimentally while using a finite correlation time.[1] An experiment on Godiva was designed to study the noise problem and to obtain experience operating on a simple low power reactor.

A. Godiva Experiment

Godiva II is a bare, fully enriched uranium, fast critical assembly at Los Alamos. It is a near-cylinder, 7.5 in. in diameter and has a critical mass of 57.7 kg.

The experimental setup is shown in Fig. 5. The input device is a $\frac{5}{16}$ in. transfer tube through the reactor center. The binary voltage command from the input channel of the punched paper tape actuates a servo valve, which allows gas to propel a one gram plastic slug either to the center of the reactor or outside the core. The reactivity of the slug is 2.5 cents, the transfer time is 5 msec, and the residence time is 45 msec.

The ion chamber output is fed to the crosscorrelator through a second order filter.

Typical results of the measurements of the impulse response of the filter and the Godiva-filter system, established for an input with $N = 1019$ and $\Delta t = 50$ msec, are shown in Fig. 6.

B. Kiwi-A3 Experiments

The encouraging results of the simulated reactor and Godiva experiments provided enough confidence in the crosscorrelation method to permit its use on Kiwi-A3.

The Kiwi-A reactors are a series of nonflying experimental prototypes for nuclear rocket engines.

[1] The noise problem is treated theoretically in the Ph.D. thesis of J. Douglas Balcomb (MIT-1961).

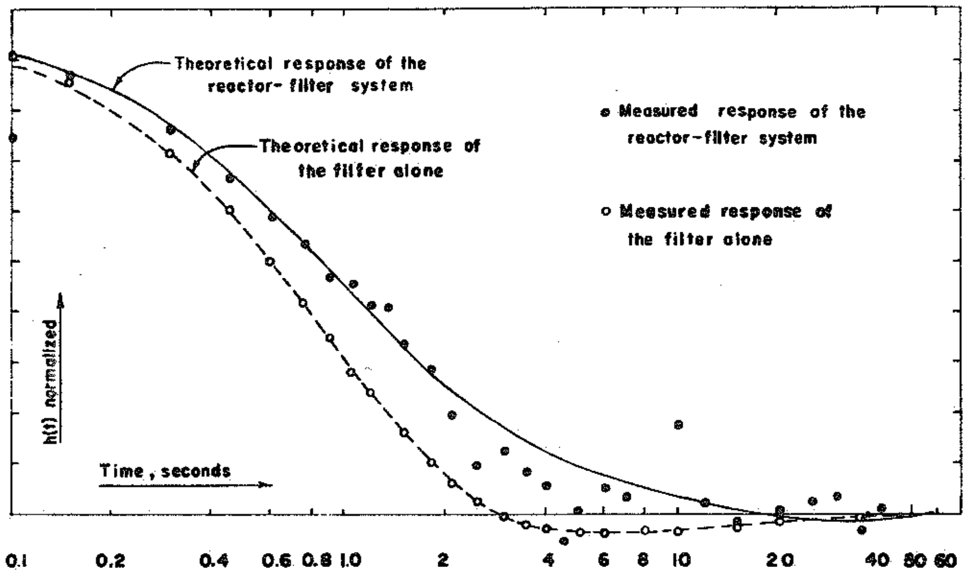

FIG. 6. Impulse responses measured by the crosscorrelation method during the Godiva experiment of 10/8/60. The data for the reactor-filter system were taken with a signal-to-noise ratio at the crosscorrelator input of about unity.

Their cores consist of uranium and graphite and are cooled by hydrogen gas. The gas is heated to a high temperature and ejected through a sonic nozzle. Kiwi-A3 is the third of the series and was successfully tested at the Nevada Test Site on October 19, 1960.

Each Kiwi-A reactor is operated for a few minutes at full power only once. During the testing at full power the reactor output power could not be perturbed more than about 1% without interfering with other measurements. The noise component of the measured reactor power is about 0.5%.

From the preceding brief description of Kiwi-A3 experimental conditions it is evident that the crosscorrelation method is the only method available for the measurement of the dynamic response of the system at full power. Step tests are excluded because of the output noise level and the limitation on allowable perturbations. Oscillation tests would take too much time.

Measurements were made at low power and at full power. The objectives of the low power measurements were to check the general performance of the neutronic instrumentation, to derive the reactor mean neutron lifetime, and to give the correlation equipment a trial run at low power. A binary input was introduced as a reactivity command of ±7 cents to the control rod servo. The system output was the current produced by an ionization chamber. Thus, the impulse response of the control rod servo, the reactor, and the neutronic instrumentation all in series was measured. The corresponding transfer function for these three series elements was calculated by taking the Laplace transform of the impulse response. To correct for the effect of the control rod servo, its impulse response was measured using

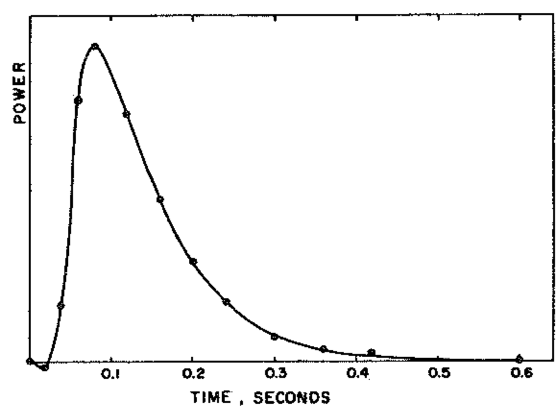

FIG. 7. Crosscorrelation method measurement of the impulse response of the Kiwi-A3 reactor at 10 kw in series with an input control rod servo system and ion chamber instrumentation.

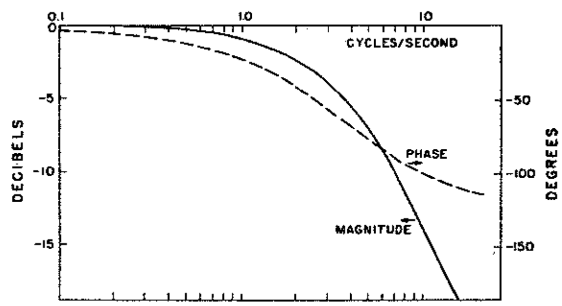

FIG. 8. Transfer function of the Kiwi-A3 reactor at 10 kw as calculated from Eq. (13).

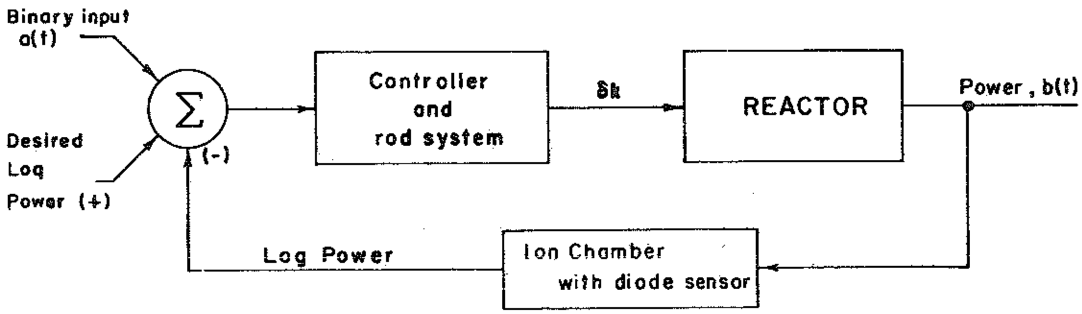

FIG. 9. Schematic diagram of the Kiwi-A3 control system. The crosscorrelation function of the input command and the measured reactor power fluctuations was calculated to determine the system impulse response.

the correlation method and its transfer function was calculated. The dynamics of the instrumentation, if operating correctly, were known. Thus, one finds the reactor transfer function from the relationship:

$$\begin{array}{c}\text{Reactor}\\ \text{transfer}\\ \text{function}\end{array} = \frac{\text{Transfer function for control rod servo, reactor and instrumentation, all in series}}{\text{Transfer function for control rod servo and instrumentation, in series}} \tag{13}$$

The combined system impulse response is shown in Fig. 7; and the reactor transfer function is shown in Fig. 8. From the break frequency in Fig. 8 it is found that the mean neutron lifetime l^* is about 4×10^{-4} sec. This value agrees fairly well with values of l^* obtained at low power by using oscillation techniques.

The experimental setup for the measurement of the system response at full power is shown in Fig. 9. The system consists of amplifiers and compensation circuits, the control rod system, the reactor, and a logarithmic detector in a feedback loop. The binary input was introduced as a command for a logarithmic power change of $\pm 2\%$ ($\Delta P/P = \Delta \log P$). The characteristics of the idealized binary chain were $N = 251$, $\Delta t = 0.02$ sec.

The control system could not follow the binary input exactly; and the maximum output power fluctuations were about $\pm 1\%$. The power fluctuations were fed to the crosscorrelator and also recorded on magnetic tape. The impulse response was measured during the reactor experiment. After the actual experiment, the impulse response was measured again by using data from the magnetic tape. The crosscorrelation experiment went off smoothly and did not interfere with the reactor operation. Typical results of the Kiwi-A3 experiments are shown in Figs. 10 and 11. This is the first time that quantitative data on the control system performance of a Kiwi reactor at full power have been obtained.

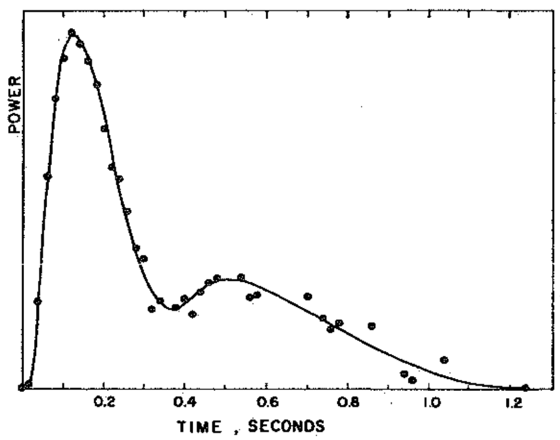

FIG. 10. Impulse response of the control loop of Fig. 9 as measured by the crosscorrelation method with the reactor at full power.

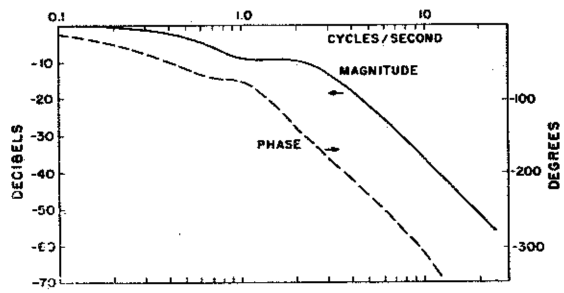

FIG. 11. Transfer function of the Kiwi-A3 power control system at full power. These curves are calculated from the data of Fig. 10 using Eq. (12).

The measurement of the response during the experiment used seven different groups of four delays each to obtain 28 points on the impulse response. The total time required to obtain data was 140 sec. Of course, if a 28 channel correlator were available, only 5 sec would be required to obtain the 28 data points.

The method used in measuring the impulse response from the magnetic tape record is outlined below. As is described in the section on Equipment, the output, the binary input, and the timing pulses are stored on magnetic tape. When the tape is played back the recorded output is fed to the crosscorrelator and the timing pulses drive the punched paper tape so that the recorded binary input and the input channel of the paper tape are synchronized. Thus, by using a large variety of delays in the delayed channels of the paper tape, one has a large number of data points available from the crosscorrelator. It should be noted that any section of the magnetic record which contains good data for at least one period, T, can be used to obtain the complete impulse response after the experiment is over.

V. CONCLUSIONS—DISCUSSION

The validity of the crosscorrelation method for the measurement of the impulse response of a nuclear reactor system has been demonstrated experimentally. The main advantages of the method are:

(a) It yields the entire information about the impulse response of the system in the shortest possible time, that is, the system settling time.

(b) The method requires only small amplitude perturbations. Consequently it is not hazardous, it is not limited by system nonlinearities, and it does not interfere with normal system operation.

(c) It can be used even in the presence of strong noise sources provided that the correlation time is increased beyond the system settling time.

The equipment described previously for the implementation of the crosscorrelation method was assembled from available parts. It is not the optimum implementation. Professional equipment is being designed to increase the number of points on the impulse response that are measured simultaneously, to make the equipment compact and portable, and to increase the measurable bandwidth. This equipment will probably use coded disk storage.

The crosscorrelation method could also be used as a continuous on-line stability monitor for a reactor system. For this purpose one introduces into the system a small amplitude input and finds the crosscorrelations for many different delays simultaneously. The points on the impulse response could be displayed on a screen and a drift toward system instability could be recognized easily from the change in the system impulse response. To minimize the amount of equipment needed in such a monitor, one could use a binary input, diode switching multiplication, and RC circuits for approximate continuous integration.

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