Thermionic Converters Operating in the Ignited Mode. Part I: Theoretical Output-Current Characteristics*

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(Received 6 December 1965; in final form 2 February 1966)

Output current characteristics of thermionic converters are derived without reference to specific elastic and inelastic collision mechanisms, and are expressed in terms of arbitrary plasma electron density and temperature profiles. The general expression for the characteristics indicates that for a given current the output voltage is composed of two parts: (a) a reference output voltage which depends only upon the output current and electrode properties and contains implicitly the effect of electron–electron collisions; and (b) a plasma voltage loss which depends only upon certain average properties of the interelectrode plasma. It is shown that conceptually different models for the interelectrode plasma can lead to similar estimates for the plasma voltage loss and to agreement with experimental output current characteristics. For this reason, it is concluded that the successful correlation of experimental output current–voltage data is necessary but not sufficient to test the validity of a given model for the interelectrode plasma.

1. INTRODUCTION

THE purpose of this paper is to investigate in a general manner the theoretical determination of output-current characteristics of thermionic converters operating in the collisional ignited mode.

Output-current characteristics have been derived in other studies.[1–3] In these studies, an appropriate set of transport equations is solved for the inhomogeneity profiles in the interelectrode plasma and then the profiles are used to compute output-current characteristics. The analyses differ from each other in the assumed ionization and recombination mechanisms and in the approximations introduced in the solution of the transport equations. In spite of the substantially different assumptions, however, a peculiar result of these studies is that they frequently lead to output-current characteristics which are in good agreement with experimental data.[1,3] This result has, understandably, created a lot of controversy regarding the nature of the predominant production, loss and transport mechanisms in the interelectrode plasma.

An important point which has been overlooked in previous studies is that output-current characteristics depend only upon certain average attributes of the profiles of electron density and temperature. In other words, it is not recognized that only a small fraction of the information contained in the detailed solutions of the plasma transport equations is relevant to the prediction of output-current characteristics.

The primary objective of this paper is to determine the features of the plasma electron density and temperature profiles which are significant in the computation of output-current characteristics. This objective is achieved by casting into an integral form the set of plasma transport differential equations presented in Ref. 4. At the plasma boundaries, the sheaths are assumed to be retarding for plasma electrons even though similar results would ensue for different boundary conditions. The analysis is valid regardless of the specific volume excitation, ionization, and recombination mechanisms.

No attempt is made here to introduce approximations and solve the plasma transport equations for the electron density and temperature profiles. This topic and numerical results and comparisons with experimental data are considered in Part II.[5]

This paper is divided into three sections as follows. In Sec. 2 the plasma transport equations are presented both in differential and integral form. In Sec. 3 output-current characteristics of thermionic converters operating in the collisional ignited mode are derived in terms of arbitrary electron density and temperature profiles. The conclusions obtained from the analysis are discussed in Sec. 4.

2. PLASMA TRANSPORT EQUATIONS

2.1. Differential Form of Transport Equations

Plasma transport differential equations for three-component, two-temperature plasmas consisting of electrons (e), ions (i), and neutrals (n) have been presented in Ref. 4. These equations are derived under assumptions which are reasonable for plasmas in thermionic converters operating in the collisional ignited mode. The equations, for a one-dimensional geometry, are repeated here for convenience:

$$dJ_e/dx = dJ_i/dx = eS \quad (1a)$$

* This work was supported principally by the National Science Foundation (Grant GK-614).

[1] H. L. Witting and E. P. Gyftopoulos, J. Appl. Phys. 36, 1328 (1965).

[2] S. Kitrilakis, A. Shavit, and N. Rasor, "The Departure of Observed Performance from the Idealized Case in Cesium Thermionic Converters," 24th Annual Conference on Physical Electronics, MIT, Cambridge, Mass., March 1964.

[3] N. S. Rasor, "Analytical Description of Cesium Diode Phenomenology," 25th Annual Conference on Physical Electronics, MIT, Cambridge, Mass., March 1965.

[4] D. R. Wilkins and E. P. Gyftopoulos, J. Appl. Phys. (to be published).

[5] D. R. Wilkins, E. P. Gyftopoulos, J. Appl. Phys. 37, 2892 (1966) (following paper).

$$dq_e/dx = -J_eE - Q_e \tag{1b}$$

$$J_e = -\mu_e\left[\frac{dp_e}{dx} + en_eE + k_e^T n_e k\frac{dT_e}{dx}\right] \tag{1c}$$

$$J_i = -\mu_i\left[\frac{dp_i}{dx} - en_iE - R_{ie} + k_i^T n_i k\frac{dT_i}{dx}\right] \tag{1d}$$

$$q_e = J_e[\tfrac{5}{2} + k_e^T]\frac{kT_e}{e} - \kappa_e\frac{dT_e}{dx}, \tag{1e}$$

where J_α, p_α, n_α, T_α, μ_α, k_α^T are the current density, pressure, particle density, temperature, mobility, and thermal diffusion ratio of species α, respectively; E is the electric field; q_e is the electron kinetic energy flux; κ_e is the electron thermal conductivity; S is the net rate of charged-particle production per unit volume; Q_e is the rate per unit volume at which electron kinetic energy is transferred to the heavy particles through collisions, R_{ie} is a force arising from the collisional transfer of directed electron momentum to the ions, and k is Boltzmann's constant.

The form of Eqs. (1) is independent of the prevailing collision laws. The elastic-collision laws affect only the values of the coefficients μ_α, k_α^T, κ_e, and the force component R_{ie}. These values can be either evaluated numerically if the collision laws are specified[4] or retained as unknown parameters to be inferred from experimental data.

The charged particle source term S and the electron-kinetic-energy transfer term Q_e depend upon the prevailing inelastic-collision mechanisms (i.e., excitation, ionization, and recombination). For given mechanisms, these terms can be expressed as functions of the particle densities and temperatures and then Eqs. (1), along with Poisson's equation, form a complete system in the variables n_e, n_i, T_e, and E.

The detailed solutions of Eqs. (1) for specified inelastic collisional processes, i.e., specified values for S and Q_e, are of importance in any description of the thermionic converter plasma itself. If the primary objective in seeking solutions, however, is the determination of output-current characteristics, then only certain average attributes of these solutions are relevant. To illustrate this point, it is convenient to rewrite Eqs. (1) in integral form.

2.2. Integral Form of Transport Equations

Integration of Eq. (1a) yields the general relation

$$J = J_e - J_i \equiv \text{const} \simeq J_e, \tag{2}$$

where J is the output-current density. The approximation of the output-current density by the electron-current density is justified because $\mu_i/\mu_e \ll 1$. In addition, Eqs. (1b) and (1c) after some elementary algebra

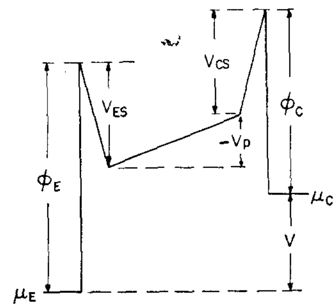

FIG. 1. Motive diagram for thermionic converters operating in the ignited mode.

may be integrated to yield

$$q_{ed} - q_{e0} = JV_P - \bar{Q}_e d;\ \bar{Q}_e d = \int_0^d Q_e(x)dx, \tag{3}$$

$$V_P = \frac{kT_{ed}}{e}\ln(J_{rd}/J) - \frac{kT_{e0}}{e}\ln(J_{r0}/J) + J\bar{\rho}_e d + V_T, \tag{4}$$

$$V_T = \int_0^d \left[(k_e^T + \tfrac{1}{2}) - \ln\frac{J_r(x)}{J}\right]\frac{k}{e}\frac{dT_e}{dx}dx;$$

$$\bar{\rho}_e d = \int_0^d \rho_e(x)dx,$$

where V_P is the voltage difference across the plasma measured from the emitter edge of the plasma, ρ_e is the electron resistivity $(1/e\mu_e n_e)$, J_r is the electron random current density $[en_e(kT_e/2\pi m_e)^{\frac{1}{2}}]$, d is the thickness of the plasma and the subscripts "0" and "d" are used to denote a quantity evaluated at the emitter and at the collector edge of the plasma, respectively.

Equations (3) and (4) are the desired integral transport equations. Because of the approximation $J \approx J_e$ it is not necessary to consider the integral of Eq. (1d).

2.3. Boundary Conditions

The boundary conditions to be satisfied by the solutions of the transport equations are obtained by writing particle and energy balances across the Debye sheaths at the edges of the plasma. The exact form of these balances depends upon the polarities of the sheaths. For example, if the sheaths are retarding for plasma electrons (see Fig. 1) and if back emission from the collector is negligible, the boundary conditions are approximately

$$J = J_E - J_{r0}\exp(-eV_{ES}/kT_{e0}) = J_{rd}\exp(-eV_{CS}/kT_{ed}), \tag{5}$$

$$q_{e0} = J_E[(2kT_E/e) + V_{ES}] - (J_E - J)[(2kT_{e0}/e) + V_{ES}], \tag{6}$$

$$q_{ed} = J[(2kT_{ed}/e) + V_{CS}], \tag{7}$$

where J_E is the emitter saturation current density, T_E is the emitter temperature, and V_{ES} and V_{CS} are the emitter and collector sheath voltage drops, respectively. It should be emphasized that Eqs. (5) through (7) represent only approximate boundary conditions since they do not account for the non-Maxwellian, anisotropic nature of the electron distribution function in the interelectrode space. Although first-order corrections which account for this effect can be introduced, the results are not sufficiently different to justify the added algebraic complexity.

Boundary conditions similar to Eqs. (5) through (7) can be derived for Debye sheaths other than those shown in Fig. 1. The conditions under which the polarities considered in Fig. 1 prevail are discussed in Part II of this study.[5]

3. OUTPUT-CURRENT CHARACTERISTICS

3.1. Derivation of Output-Current Characteristics

For thermionic converters in which the Debye sheath polarities are as shown in Fig. 1, the output voltage V is given by the relation

$$V=(\phi_E-\phi_C)/e+V_{CS}-V_{ES}-V_P \equiv V_0+V_{CS}-V_{ES}-V_P, \quad (8)$$

where ϕ_E and ϕ_C are the emitter and collector work functions, respectively, and $V_0\equiv(\phi_E-\phi_C)/e$, i.e., the contact potential.

Equations (3) through (8) may be combined to yield output-current characteristics in terms of arbitrary electron density and temperature profiles. The characteristics can be written in several convenient forms. Two of these forms are

$$V=V_0+(kT_{e0}/e)\ln(J_E/J-1)-J\bar{\rho}_e d-V_T, \quad (9)$$

or

$$V=V_0+\frac{kT_E}{e}\frac{\ln(J_E/J-1)}{1+J\ln(J_E/J-1)/2J_E}-V_1, \quad (10)$$

$$V_1=\frac{J\bar{\rho}_e d+J\ln(J_E/J-1)[\bar{Q}_e d/J+2k(T_{ed}-T_{e0})/e]/2J_E+V_T}{1+J\ln(J_E/J-1)/2J_E}. \quad (11)$$

Equations (9) and (10) are valid regardless of the prevaling inelastic collision mechanisms. The mechanisms specify the exact shape of the electron density and temperature profiles. Even without knowledge of these profiles, however, several features of the output-current characteristics can be deduced.

3.2. Interpretation of Output-Current Characteristics

It is apparent from Eqs. (9) and (10) that the output-current characteristics of thermionic converters operating in the collisional ignited mode depend only on certain average attributes of the plasma electron density and temperature profiles. More specifically, consider first Eq. (9). Suppose, for example, that the electron temperature is adequately represented by an average value $\bar{T}_e$ throughout the plasma so that

$$V=V_0+(k\bar{T}_e/e)\ln(J_E/J-1)-J\rho_e^\infty d-(Jd/e\mu_e^0)\langle 1/n_e\rangle_{\text{av}}, \quad (9a)$$

where ρ_e^∞ is the Coulomb resistivity, μ_e^0 is the electron mobility in the absence of Coulomb collisions, $\langle 1/n_e\rangle_{\text{av}}=(1/d)\int_0^d[1/n_e(x)]dx$, and the approximation $\bar{\rho}_e\simeq\rho_e^\infty+(1/e\mu_e^0)\langle 1/n_e\rangle_{\text{av}}$ has been introduced.[4] Then the output-current characteristics depend only on the average electron temperature and on the average inverse electron density.

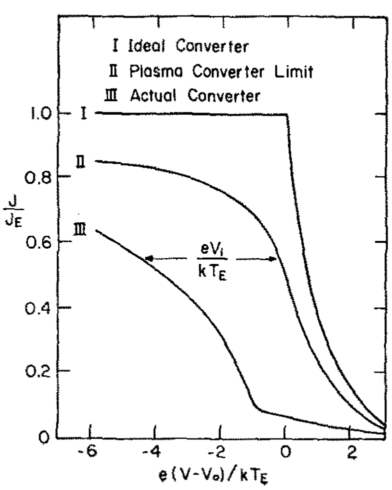

FIG. 2. Comparison of output-current characteristics for different operating conditions.

Equation (9a) is applicable to previous studies based on a two-step molecular ion-formation mechanism[1] and on a single-step atomic ion-formation mechanism[3], where electron temperature gradients are neglected. In these studies similar numerical estimates for $\bar{T}_e$ and $\langle 1/n_e\rangle_{\text{av}}$ are obtained. Hence, it is not surprising that the derived output-current characteristics in both Refs. 1 and 3 are similar. However, the fact that these characteristics are in good agreement with experimental data cannot be considered as conclusive regarding whether molecular or atomic ions are formed in the interelectrode space. Parallel observations can be made about other analyses.

Next consider Eq. (10). Suppose again that the electron temperature is uniform. Then the voltage loss V_1 [Eq. (11)] and consequently the output-current characteristics depend only on the average resistivity $\bar{\rho}_e$ and the average kinetic energy transfer $\bar{Q}_e$.

In Ref. 1, the kinetic energy transfer term is found to be small compared to the resistive terms. In Ref. 3, the kinetic-energy transfer term is neglected altogether. Thus, the derived output-current characteristics in references 1 and 3 depend only on $\langle 1/n_e\rangle_{av}$ for which similar values are computed. It is evident again that the successful prediction of output-current characteristics is not sufficient to test the validity of a given plasma model. A much more meaningful test is the prediction of electron density and temperature profiles and their comparison with similar measurements, as discussed in Part II.[5]

Another feature of Eq. (10) is deduced by assuming that there are no electron–heavy particle elastic and inelastic collisions and that the electron temperature is uniform. Then $V_1=0$ and Eq. (10) yields a reference output-current characteristic which depends explicitly only upon the electrode properties and implicitly on the effect of electron–electron collisions. This characteristic is shown in Fig. 2 and compared to that of the ideal thermionic converter with identical electrode properties. It is clear that electron–electron collisions reduce the performance of thermionic converters because electrons are scattered back to the emitter. When $V_1\neq 0$, i.e., when there are electron–heavy particle elastic and inelastic collisions and temperature gradients in the plasma, the performance of the converter is further reduced as schematically shown in Fig. 2. Thus, Eq. (10) indicates that output-current characteristics may be interpreted in terms of a reference characteristic which depends only upon the electrode properties and electron–electron collisions, and a plasma voltage loss which depends only upon the electron–heavy particle physics. Experimental data have been successfully interpreted in this manner in the past.[2]

Finally, the effects of the different physical processes affecting the current flow through a thermionic converter can conveniently be brought forth by rewriting Eq. (9) in the form

$$\frac{J_E-J}{J}=\exp\left[\frac{e(V-V_0)}{kT_{e0}}\right]\exp\left[\frac{eJd\bar{\rho}_e}{kT_{e0}}\right]\exp\left[\frac{eV_T}{kT_{e0}}\right]. \quad (12)$$

In other words, the ratio of the current J_E-J returned to the emitter to the current J reaching the collector is influenced by three factors. These factors account for the return of electrons to the emitter due to: (a) the electric field, (b) electron–heavy particle momentum-transfer collisions in the plasma, and (c) diffusion along the electron temperature gradient. Implicit in all these factors are, of course, electron–electron collisions which transform the half-Maxwellian distribution of the thermionically emitted electrons into the near-Maxwellian distribution computed for the derivation of Eqs. (1).[4]

4. CONCLUSIONS

General output-current characteristics of thermionic converters operating in the collisional ignited mode are derived without reference to specific elastic and inelastic collision mechanisms. These characteristics depend explicitly only upon certain average attributes of the plasma electron density and temperature profiles. Since these attributes may be numerically similar for conceptually different plasma models, it is concluded that the successful correlation of experimental output-current characteristics is not a sufficient test of the validity of a given plasma model.

Reference output-current characteristics of thermionic converters without electron–heavy particle collisions and electron temperature gradients are also derived. These characteristics depend explicitly only on the electrode properties and implicitly on the effects of electron–electron collisions. They represent an upper performance limit for plasma thermionic converters. The limit is lower than that of ideal thermionic converters.

Experimental output-current characteristics may be interpreted in terms of a departure from the above reference characteristics. The measure of the departure is the voltage V_1[Eq (11)] due to resistive losses, collisional-electron kinetic-energy transfer losses and plasma-electron temperature gradients.

ACKNOWLEDGMENT

The first author gratefully acknowledges a fellowship granted by the Thompson–Ramo–Wooldridge Space Technology Laboratories.