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Transport Phenomena in Low-Energy Plasmas*†

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An extended Chapman–Cowling formalism is used to derive transport differential equations for the charged-particle species in low-energy, three-component, two-temperature plasmas consisting of electrons, ions, and neutrals. The analysis is developed in two parts: (a) the charged-particle Boltzmann equations are solved for the charged-particle distribution functions; (b) these distribution functions are used to reduce the charged-particle hydrodynamic equations into a system of differential equations involving the densities, temperatures, and electrostatic potential of the plasma, and a set of transport coefficients which depend upon the prevailing collision laws. A general procedure for the computation of the transport coefficients is given, and illustrative numerical results are presented for specific collision laws.

1. INTRODUCTION

THE purpose of this paper is to derive a set of plasma transport differential equations suitable for the analysis of transport phenomena in low-energy, three-component, two-temperature plasmas.

Transport differential equations for nonuniform plasmas have been derived by others.[1–4] The resulting equations are expressed in terms of the densities, temperatures, and electric field of the plasma, and contain a set of transport coefficients which depend upon the prevailing collision laws. Chapman and Cowling[3] have considered the case of single-temperature plasmas in which inelastic collisional processes may be neglected. Stanchanov and Stepanov[4] have recently modified the Chapman–Cowling formalism to include two-temperature plasmas. These authors, however, have presented the transport coefficients in a form which is not amenable to physical interpretation, and have imposed unnecessary restrictions on the elastic collision laws. None of the previous authors have discussed in any detail the effect of inelastic collisions.

In this paper the Chapman–Cowling formalism is extended to obtain transport equations for the charged-particle species in low-energy, three-component, two-temperature plasmas. The derived equations are novel in that they take into account both charged-particle and charged-particle–neutral-particle elastic collisions, inelastic collisions, and electron kinetic energy transport. The equations may be solved to yield the charged-particle density, electron temperature, and electric field profiles within the plasma. These profiles may in turn be used to compute current–voltage characteristics and other properties of a plasma device.[5]

In the derivations no restrictions are imposed on the elastic collision laws. Inelastic collisions are included through appropriate approximations. The analysis is applicable to plasmas in which: (a) the mass of the negative charge carriers is much smaller than the mass of positive ions and of neutral particles; (b) the neutral-ion collision frequency is much smaller than the neutral–neutral collision frequency; (c) any characteristic mean free path is much smaller than the characteristic size of the plasma; and (d) the drift velocities of all species are much smaller than the corresponding random velocities.

The paper is divided into five sections. In Sec. 2 the Boltzmann and macroscopic hydrodynamic equations which describe the charged-particle motions are presented. In Sec. 3 these equations are adapted to the plasmas described above and the Boltzmann equations are solved for the charged-particle distribution functions. In Sec. 4 the distribution functions are used to reduce the charged-particle hydrodynamic equations into a set of differential equations involving the densities, temperatures, and electric field of the plasma, and a set of transport coefficients which depend upon the prevailing collision laws. The specific procedure for the computation of the transport coefficients is also given. In Sec. 5 the evaluation of the transport coefficients is illustrated through several practical examples.

2. BOLTZMANN AND HYDRODYNAMIC EQUATIONS

2.1. Boltzmann Equations

The Boltzmann equations for electrons and ions in a steady-state, three-component plasma consisting of electrons (e), monatomic ions (i), and monatomic neutrals (o), may be written as

$$\mathbf{v}_\alpha\cdot\nabla_r f_\alpha+\frac{e_\alpha\mathbf{E}}{m_\alpha}\cdot\nabla_{v\alpha} f_\alpha=\sum_\beta J_{\alpha\beta}(f_\alpha,f_\beta)+\left(\frac{\partial f_\alpha}{\partial t}\right)_{in};\quad(1)$$

$$\alpha=e,\,i;\quad \beta=e,\,i,\,o,$$

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where e_α, m_α, $\mathbf{v}_\alpha$, and f_α are the charge, mass, velocity, and distribution function of species α, respectively; $\mathbf{E}$ is the electric field; $J_{\alpha\beta}(f_\alpha,f_\beta)$ is the elastic collision integral for collisions between species α and β; and $(\partial f_\alpha/\partial t)_{in}$ is the inelastic collision integral for species α. For short-range, two-body interactions, the elastic collision integral $J_{\alpha\beta}(f_\alpha,f_\beta)$ may be written in the Boltzmann form[3]:

$$J_{\alpha\beta}(f_\alpha,f_\beta)=\int\int(f_\alpha'f_\beta'-f_\alpha f_\beta)\times\rho_{\alpha\beta}(|\mathbf{v}_\alpha-\mathbf{v}_\beta|,x)d^2\Omega d^3v_\beta, \quad (2)$$

where $\rho_{\alpha\beta}\equiv|\mathbf{v}_\alpha-\mathbf{v}_\beta|\sigma_{\alpha\beta}(|\mathbf{v}_\alpha-\mathbf{v}_\beta|,x)$, $\sigma_{\alpha\beta}(|\mathbf{v}_\alpha-\mathbf{v}_\beta|,x)$ is the differential elastic scattering cross section, x is the polar scattering angle in the center of mass coordinates, $d^2\Omega$ is an element of solid angle, and $f_\alpha'=f_\alpha(\mathbf{r},\mathbf{v}_\alpha')$ where $\mathbf{v}_\alpha'$ is the velocity prior to collision. In writing Eq. (2) for like-particle collisions, the second subscript is omitted to avoid confusion concerning the variables of integration.

Note that the set of Eqs. (1) does not include a Boltzmann equation for neutral particles. The reason is that, if the analysis is restricted to plasmas in which the neutral–ion collision frequency is much smaller than the neutral–neutral collision frequency, then the distribution function of the neutrals is essentially unperturbed by the motions of the charged particles. Hence, the neutral distribution function need only be determined to zero order:

$$f_0(\mathbf{r},\mathbf{v}_0)\equiv f_0^0(\mathbf{r},\mathbf{v}_0)\equiv n_0(m_0/2\pi kT_0)^{\frac{3}{2}}\times\exp(-m_0v_0^2/2kT_0), \quad (3)$$

where n_α and T_α are the density and temperature of species α, respectively, and k is the Boltzmann constant.

2.2. Hydrodynamic Equations

The macroscopic mass, momentum, and energy conservation equations for species $\alpha(\alpha=e,i)$ may be derived directly from Eqs. (1). To this end, multiply each equation by 1, $m_\alpha\mathbf{v}_\alpha$, $m_\alpha\mathbf{v}_\alpha^2/2$, and integrate each result over the α-velocity space. Thus:

$$\nabla_r\cdot\mathbf{\Gamma}_\alpha=S_\alpha^{in}, \quad (4a)$$

$$\nabla_r p_\alpha+\nabla_r\overline{\overline{\pi}}_\alpha-e_\alpha n_\alpha\mathbf{E}-\sum_{\beta\neq\alpha}\mathbf{R}_{\alpha\beta}-\mathbf{R}_\alpha^{in}=0, \quad (4b)$$

$$\nabla_r\cdot\mathbf{q}_\alpha=e_\alpha\mathbf{\Gamma}_\alpha\cdot\mathbf{E}-\sum_{\beta\neq\alpha}Q_{\alpha\beta}-Q_\alpha^{in}, \quad (4c)$$

where

$$n_\alpha\equiv\int f_\alpha d^3v_\alpha,\quad p_\alpha\equiv\int(m_\alpha v_\alpha^2/3)f_\alpha d^3v_\alpha,\quad \mathbf{\Gamma}_\alpha\equiv\int\mathbf{v}_\alpha f_\alpha d^3v_\alpha,\quad \mathbf{q}_\alpha\equiv\int(m_\alpha v_\alpha^2/2)\mathbf{v}_\alpha f_\alpha d^3v_\alpha,$$

$$\pi_{\alpha jk}\equiv\int m_\alpha(v_{\alpha j}v_{\alpha k}-v_\alpha^2\delta_{jk}/3)f_\alpha d^3v_\alpha,\quad \mathbf{R}_{\alpha\beta}\equiv\int m_\alpha\mathbf{v}_\alpha J_{\alpha\beta}(f_\alpha,f_\beta)d^3v_\alpha,\quad Q_{\alpha\beta}\equiv-\int\frac{m_\alpha v_\alpha^2}{2}J_{\alpha\beta}(f_\alpha,f_\beta)d^3v_\alpha, \quad (5)$$

$$S_\alpha^{in}\equiv\int\left(\frac{\partial f_\alpha}{\partial t}\right)_{in}d^3v_\alpha,\quad \mathbf{R}_\alpha^{in}\equiv\int m_\alpha\mathbf{v}_\alpha\left(\frac{\partial f_\alpha}{\partial t}\right)_{in}d^3v_\alpha,\quad Q_\alpha^{in}\equiv-\int\frac{m_\alpha v_\alpha^2}{2}\left(\frac{\partial f_\alpha}{\partial t}\right)_{in}d^3v_\alpha. \quad (6)$$

In the above equations and identities, p_α, $\mathbf{\Gamma}_\alpha$, $\mathbf{q}_\alpha$, and $\pi_{\alpha jk}$ are the pressure, particle current, kinetic energy flux, and jkth component of the viscosity tensor, respectively, for species α. Also, $\mathbf{R}_{\alpha\beta}$ and $Q_{\alpha\beta}$ are the rates of elastic momentum and energy transfer, respectively, between species α and β; it is easily shown that $\mathbf{R}_{\alpha\beta}\equiv-\mathbf{R}_{\beta\alpha}$ and $Q_{\alpha\beta}\equiv-Q_{\beta\alpha}$.[3] Finally, S_α^{in}, $\mathbf{R}_\alpha^{in}$, and Q_α^{in} are the inelastic rates of production of particles, momentum, and energy, respectively, for species α. The integrals involved in Identities (5) can be evaluated if the charged particle distribution functions are known. If the distribution functions are expressed in terms of n_α, T_α, and $\mathbf{E}$, Eqs. (4) may be reduced to a system of plasma transport differential equations in these same variables. The determination of the distribution functions in terms of n_α, T_α, and $\mathbf{E}$ is considered in Sec. 3, and the plasma transport differential equations are derived in Sec. 4.

3. SOLUTION OF BOLTZMANN EQUATIONS

3.1. Properties of Plasmas of Interest

The plasmas of interest here have certain characteristic properties which, when reflected in Eqs. (1), lead to mathematical simplifications and thereby to explicit solutions of those equations. These properties and the related mathematical simplifications are discussed below.

Near-Maxwellian Property: For plasmas which are collision dominated, the charged-particle distribution functions are nearly Maxwellian. Therefore, it is reasonable to seek perturbed distribution functions of the form:

$$f_\alpha(\mathbf{r},\mathbf{v}_\alpha)\equiv f_\alpha^0(\mathbf{r},\mathbf{v}_\alpha)[1+\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)];\ \phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)\ll1;\ \alpha=e,i,$$
$$f_\alpha^0(\mathbf{r},\mathbf{v}_\alpha)\equiv n_\alpha(m_\alpha/2\pi kT_\alpha)^{\frac{3}{2}}\exp(-m_\alpha v_\alpha^2/2kT_\alpha). \quad (7)$$

The meaning of the inequality $\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)\ll1$ is that the nonlinear Boltzmann Eqs. (1) may be linearized. It is shown subsequently that the scalar perturbation functions $\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)$ are of the form

$$\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)=\boldsymbol{\omega}_\alpha(\mathbf{r},v_\alpha)\cdot\mathbf{v}_\alpha/v_\alpha, \tag{8}$$

where the vector $\boldsymbol{\omega}_\alpha(\mathbf{r},v_\alpha)$ is linear in the density, temperature, and electrostatic potential gradients and depends only upon $|\mathbf{v}_\alpha|$. Note that for distribution functions of the form indicated by Eqs. (7) and (8) the densities, temperatures, and pressures are defined in a manner which is consistent with the usual kinetic theory definitions:

$$n_\alpha=\int f_\alpha d^3v_\alpha;\quad 3n_\alpha kT_\alpha=\int m_\alpha v_\alpha^2 f_\alpha d^3v_\alpha;\quad p_\alpha=n_\alpha kT_\alpha.$$

Small m_e/m_β Property: For plasmas in which $m_e/m_\beta\ll 1$ ($\beta=i,\ o$), the elastic collision integrals J_{ei}, J_{eo}, and J_{ie} of Eqs. (2) may be simplified by neglecting terms of the order $(m_e/m_\beta)^{\frac{1}{2}}$. Specifically,

$$J_{e\beta}(f_e,f_\beta)\simeq-\nu_{e\beta}(v_e)f_e^0\phi_e;\quad \beta=i,\ o, \tag{9}$$

$$J_{ie}(f_i,f_e)\simeq(m_e/p_i)f_i^0\mathbf{v}_i\cdot\int f_e\mathbf{v}_e\nu_{ei}(v_e)d^3v_e, \tag{10}$$

where

$$\nu_{e\beta}(v_e)\equiv n_\beta v_e\int(1-\cos x)\sigma_{e\beta}(v_e,x)d^2\Omega;\quad \beta=i,\ o.$$

The end result of approximations (9) and (10) is to decouple the electron and ion Boltzmann equations in velocity space. A further consequence of these approximations is that the electron–heavy-particle energy exchanges reduce to zero, i.e.,

$$Q_{ei}=Q_{ie}=Q_{eo}=0. \tag{11}$$

$T_i=T_o$ Property: Since the collisional coupling between the heavy-particle species is efficient, it may be assumed that the heavy-particle temperatures are equal, i.e., $T_i=T_o$. Because of this approximation it is no longer necessary to retain the ion energy conservation equation [Eq. (4c); $\alpha=i$].

Low-Energy Property: For low-energy plasmas, the inelastic collision integrals $(\partial f_\alpha/\partial t)_{in}$ can be neglected when solving Eqs. (1) for the distribution functions. The electron distribution function so obtained will be valid at least for electron energies less than the lowest excitation energy of the neutral species, and may be used to evaluate (at least) those macroscopic quantities which are insensitive to its high-energy tail. The electron distribution function will also be valid for higher electron energies provided electron–electron collisions populate the high-energy tail rapidly compared to the net rate at which it is depleted through inelastic collisions. This latter requirement is fulfilled, for example, in plasmas which are approximately in local thermodynamic equilibrium so that forward and reverse reactions (e.g., excitation and de-excitation) proceed at approximately the same rate. Arguments similar to the preceding can be presented concerning the ion distribution function.

Consistent with the omission of the terms $(\partial f_\alpha/\partial t)_{in}$ in Eqs. (1) is also the omission of the inelastic collision forces $\mathbf{R}_\alpha^{in}$ in Eqs. (4b):

$$\mathbf{R}_\alpha^{in}=0;\quad \alpha=e,\ i. \tag{12}$$

The inelastic source terms in S_α^{in} in Eqs. (4a) and the inelastic energy transfer term Q_e^{in} in Eq. (4c); ($\alpha=e$) are retained, however, since the corresponding terms arising from elastic collisions are zero.

3.2. Linearized Boltzmann Equations

As stated previously, the charged-particle Boltzmann Eqs. (1) may be linearized for the analysis of near-Maxwellian plasmas. The linearized form is obtained by substituting the distribution functions [Eqs. (7)] into Eqs. (1), by using the approximations discussed above, and by neglecting second-order terms in $\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)$. Thus:

$$(1/p_e)f_e^0[\mathbf{F}_e+(u_e^2-\tfrac{5}{2})n_ek\nabla_rT_e]\cdot\mathbf{v}_e$$
$$=-[\nu_{ei}(v_e)+\nu_{eo}(v_e)]f_e\phi_e-n_eI_e(\phi_e), \tag{13}$$

$$(1/p_i)f_i^0[\mathbf{F}_i+(u_i^2-\tfrac{5}{2})n_ik\nabla_rT_i]\cdot\mathbf{v}_i$$
$$=-n_i^2I_i(\phi_i)-n_in_oI_{io}(\phi_i), \tag{14}$$

where

$$\mathbf{u}_\alpha=(m_\alpha/2kT_\alpha)^{\frac{1}{2}}\mathbf{v}_\alpha,\qquad \mathbf{F}_e\equiv\nabla_rp_e+en_e\mathbf{E},$$

$$\mathbf{F}_i\equiv\nabla_rp_i-en_i\mathbf{E}-\mathbf{R}_{ie},\quad \mathbf{R}_{ie}\equiv m_e\int f_e\mathbf{v}_e\nu_{ei}(v_e)d^3v_e,$$

and the linear integral operators I_α and $I_{\alpha\beta}$ are defined by:

$$I_\alpha[G(\mathbf{v}_\alpha)]\equiv\frac{1}{n_\alpha^2}\int\int f_\alpha^0(\mathbf{r},\mathbf{v}_\alpha)f_\alpha^0(\mathbf{r},\mathbf{v})$$
$$\cdot[G(\mathbf{v}_\alpha)+G(\mathbf{v})-G(\mathbf{v}_\alpha')-G(\mathbf{v}')]$$
$$\times p_\alpha(|\mathbf{v}_\alpha-\mathbf{v}|,x)d^2\Omega d^3v,$$

$$I_{\alpha\beta}[G(\mathbf{v}_\alpha)]\equiv\frac{1}{n_\alpha n_\beta}\int\int f_\alpha^0(\mathbf{r},\mathbf{v}_\alpha)f_\beta^0(\mathbf{r},\mathbf{v}_\beta)$$
$$\cdot[G(\mathbf{v}_\alpha)-G(\mathbf{v}_\alpha')]\rho_{\alpha\beta}(|\mathbf{v}_\alpha-\mathbf{v}_\beta|,x)d^2\Omega d^3v_\beta,$$

where $G(\mathbf{v}_\alpha)$ may be any scalar or vector function of $\mathbf{v}_\alpha$. Note that the definition of $\mathbf{F}_e$ does not include a term of the form $\mathbf{R}_{ei}$. Also note that the operators I_α and $I_{\alpha\beta}$ satisfy the symmetry relation[3]:

$$\int H(\mathbf{v}_\alpha)\cdot I[G(\mathbf{v}_\alpha)]d^3v_\alpha=\int G(\mathbf{v}_\alpha)\cdot I[H(\mathbf{v}_\alpha)]d^3v_\alpha;$$
$$I=I_\alpha,\ I_{\alpha\beta}, \tag{15}$$

where $H(\mathbf{v}_\alpha)$ is any scalar or vector function of $\mathbf{v}_\alpha$.

The nature of the driving terms in the linear integral Eqs. (13)–(14) permits one to seek general solutions of the form[3]

$$\phi_\alpha(\mathbf{r},\mathbf{v}_\alpha)=-\frac{1}{n_\alpha p_\alpha}\left(\frac{2kT_\alpha}{m_\alpha}\right)^{\frac{1}{2}}$$
$$\times[A_\alpha(u_\alpha)\mathbf{F}_\alpha+B_\alpha(u_\alpha)n_\alpha k\nabla_r T_\alpha]\cdot\frac{\mathbf{u}_\alpha}{u_\alpha};\quad \alpha=e,\,i,\qquad(16)$$

where $A_\alpha(u_\alpha)$ and $B_\alpha(u_\alpha)$ are scalar functions of the magnitude of $\mathbf{u}_\alpha$. Substitution of Eqs. (16) into Eqs. (13), (14) yields

$$\frac{1}{n_e}f_e^0\mathbf{u}_e=I_e(\mathbf{A}_e)+\frac{1}{n_e^2}f_e^0[\nu_{ei}(v_e)+\nu_{eo}(v_e)]\mathbf{A}_e,\qquad(17)$$

$$\frac{1}{n_e}(u_e^2-\tfrac{5}{2})f_e^0\mathbf{u}_e=I_e(\mathbf{B}_e)+\frac{1}{n_e^2}f_e^0[\nu_{ei}(v_e)+\nu_{eo}(v_e)]\mathbf{B}_e,\qquad(18)$$

$$\frac{1}{n_i}f_i^0\mathbf{u}_i=I_i(\mathbf{A}_i)+\frac{n_0}{n_i}I_{io}(\mathbf{A}_i),\qquad(19)$$

$$\frac{1}{n_i}(u_i^2-\tfrac{5}{2})f_i^0\mathbf{u}_i=I_i(\mathbf{B}_i)+\frac{n_0}{n_i}I_{io}(\mathbf{B}_i),\qquad(20)$$

where $\mathbf{A}_\alpha\equiv A_\alpha(u_\alpha)\mathbf{u}_\alpha/u_\alpha$ and $\mathbf{B}_\alpha\equiv B_\alpha(u_\alpha)\mathbf{u}_\alpha/u_\alpha$.

The meaning of Eqs. (17)–(20) is that the problem of determining the distribution functions, $f_\alpha(\mathbf{r},\mathbf{v}_\alpha);\alpha=e,\,i,$ is reduced to that of finding four scalar functions $A_e(u_e)$, $B_e(u_e)$, $A_i(u_i)$, and $B_i(u_i)$ which are solutions of Eqs. (17)–(20), respectively. Note that if electron–electron collisions are negligible, Eqs. (17) and (18) may be readily solved for the scalar functions $A_e(u_e)$ and $B_e(u_e)$ and the electron distribution function follows directly. This solution corresponds to the important case of a Lorentz plasma and is discussed in detail in Sec. 5. The general solution of Eqs. (17)–(20) cannot be obtained exactly and hence it is necessary to resort to techniques for generating approximate solutions. One such technique involves the expansion of $A_\alpha(u_\alpha)$ and $B_\alpha(u_\alpha)$ into series of orthonormal functions and is discussed below.

3.3. Sonine Polynomial Expansion for $A_\alpha(u_\alpha)$ and $B_\alpha(u_\alpha)$

In seeking approximate solutions to Eqs. (17)–(20), it is expedient to expand the scalar functions $A_\alpha(u_\alpha)/u_\alpha$ and $B_\alpha(u_\alpha)/u_\alpha$ into series of the complete, orthonormal Sonine polynomials of order $\frac{3}{2}$, i.e.:

$$A_\alpha(u_\alpha)/u_\alpha=\sum_{n=0}^{\infty}a_\alpha^{(n)}S_n^{\frac{3}{2}}(u_\alpha^2);$$
$$B_\alpha(u_\alpha)/u_\alpha=\sum_{n=0}^{\infty}b_\alpha^{(n)}S_n^{\frac{3}{2}}(u_\alpha^2),\qquad(21)$$

where $a_\alpha^{(n)}$ and $b_\alpha^{(n)}$ are exapnsion coefficients and $S_n^{\frac{3}{2}}(z)$ is a Sonine polynomial of order $\frac{3}{2}$ and argument z.[3]

Substitution of these series into Eqs. (17)–(20), dot-multiplication of the results by $S_m^{\frac{3}{2}}(u_\alpha^2)\mathbf{u}_\alpha$, and integration over velocity space, yields four infinite sets of linear algebraic equations of the form:

$$\sum_{n=0}^{\infty}\delta_\alpha^{(m,n)}a_\alpha^{(n)}=\beta_\alpha^{(m)},\qquad(22a)$$

$$\sum_{n=0}^{\infty}\delta_\alpha^{(m,n)}b_\alpha^{(n)}=\gamma_\alpha^{(m)};\quad m=0,1,\cdots\infty;\quad\alpha=e,\,i,\qquad(22b)$$

where

$$\delta_e^{(m,n)}\equiv\int S_m^{\frac{3}{2}}(u_e^2)\mathbf{u}_e\cdot I_e[S_n^{\frac{3}{2}}(u_e^2)\mathbf{u}_e]d^3v_e$$
$$+(1/n_e^2)\int f_e^0u_e^2[\nu_{ei}(v_e)+\nu_{eo}(v_e)]\times S_m^{\frac{3}{2}}(u_e^2)S_n^{\frac{3}{2}}(u_e^2)d^3v_e,$$

$$\delta_i^{(m,n)}\equiv\int S_m^{\frac{3}{2}}(u_i^2)\mathbf{u}_i\cdot I_i[S_n^{\frac{3}{2}}(u_i^2)\mathbf{u}_i]d^3v_i$$
$$+(n_o/n_i)\int S_m^{\frac{3}{2}}(u_i^2)\mathbf{u}_i\cdot I_{io}[S_n^{\frac{3}{2}}(u_i^2)\mathbf{u}_i]d^3v_i,$$

$$\beta_\alpha^{(m)}\equiv\frac{1}{n_\alpha}\int S_m^{\frac{3}{2}}(u_\alpha^2)u_\alpha^2f_\alpha^0d^3v_\alpha=\begin{cases}\frac{3}{2}, & m=0;\\ 0, & m\neq0;\quad\alpha=e,\,i,\end{cases}$$

$$\gamma_\alpha^{(m)}\equiv\frac{1}{n_\alpha}\int S_m^{\frac{3}{2}}(u_\alpha^2)u_\alpha^2(u_\alpha^2-\tfrac{5}{2})f_\alpha^0d^3v_\alpha=\begin{cases}-15/4, & m=1;\\ 0, & m\neq1;\quad\alpha=e,\,i.\end{cases}$$

Note that the relations $\beta_\alpha^{(m)}=0$ for $m\neq0$, and $\gamma_\alpha^{(m)}=0$ for $m\neq1$ are a consequence of the use of Sonine polynomials of order $\frac{3}{2}$. By virtue of Eqs. (22) the problem of solving Eqs. (17)–(20) for the scalar functions $A_\alpha(u_\alpha)$ and $B_\alpha(u_\alpha)$ ($\alpha=e,\,i$) is reduced to that of solving four infinite sets of algebraic equations for the Sonine expansion coefficients $a_\alpha^{(n)}$ and $b_\alpha^{(n)}$. Approximate solutions to any desired degree of accuracy are obtained by truncating the Sonine expansions after N terms and solving the resulting $4N$ equations. The matrix elements $\delta_\alpha^{(m,n)}$ may, in principle, be evaluated once the collision laws are specified. More specifically, both terms in the expression for $\delta_i^{(m,n)}$ and the first term in the expression for $\delta_e^{(m,n)}$ are special cases of a general set of collision integrals which have been evaluated and tabulated by Chapman and Cowling[3] and Hirshfelder *et al.*[2] The remaining terms in the expression for $\delta_e^{(m,n)}$ may be evaluated by straightforward integration. From the symmetry property of the I_α and $I_{\alpha\beta}$ operators [Eq. (15)], it is apparent that $\delta_\alpha^{(m,n)}=\delta_\alpha^{(n,m)};\alpha=e,\,i.$

Equations (22) for $m=0$ are equivalent to the macroscopic momentum conservation equations [Eqs. (4b)] provided the second-order term $\nabla_r\cdot\overline{\overline{\pi}}$, and the inelastic collision forces $\mathbf{R}_\alpha{}^{in}$ are neglected [see Eq. (12)]. Consequently, the momentum conservation Eqs. (4b) are satisfied in determining the distribution functions and need not be considered further in discussing the hydrodynamic equations. In addition, it can be shown that the linear algebraic Eqs. (22) for $m>0$ correspond to moments of the Boltzmann equations higher than that of the energy conservation Eqs. (4c).

4. PLASMA TRANSPORT DIFFERENTIAL EQUATIONS

4.1. Reduced Set of Hydrodynamic Equations

As a consequence of the assumptions and results of Sec. 3, the set of hydrodynamic Eqs. (4) may be replaced by a reduced set of hydrodynamic equations in which the macroscopic momentum and the ion energy equations do not appear. The new set is

$$\nabla_r\cdot\mathbf{J}_e=eS_e{}^{in};\quad \nabla_r\cdot\mathbf{J}_i=eS_i{}^{in}, \tag{23a}$$

$$\nabla_r\cdot\mathbf{q}_e=-\mathbf{J}_e\cdot\mathbf{E}-Q_e{}^{in}, \tag{23b}$$

where $\mathbf{J}_\alpha=e\mathbf{\Gamma}_\alpha$, and e is the electronic charge. Equations (23) can be written in terms of the plasma densities, temperatures, and electric field and a set of transport coefficients as shown below.

4.2. Particle Currents and Electron Kinetic Energy Flux

The particle currents $\mathbf{J}_e$ and $\mathbf{J}_i$, and the electron kinetic energy flux $\mathbf{q}_e$, may be determined from the distribution functions of Sec. 3. Indeed:

$$\mathbf{J}_e\equiv e\int \mathbf{v}_e f_e d^3v_e=-\mu_e[\nabla_r p_e+en_e\mathbf{E}+k_e{}^Tn_ek\nabla_rT_e], \tag{24a}$$

$$\mathbf{J}_i\equiv e\int \mathbf{v}_i f_i d^3v_i=-\mu_i[\nabla_r p_i-en_i\mathbf{E}-\mathbf{R}_{ie}+k_i{}^Tn_ik\nabla_rT_i], \tag{24b}$$

$$\mathbf{q}_e\equiv\int (m_ev_e{}^2/2)\mathbf{v}_ef_ed^3v_e=\frac{J_e}{e}\frac{5}{2}kT_e+\frac{J_e}{e}k_e{}^TkT_e-\kappa_e\nabla_rT_e, \tag{25}$$

where μ_α and $k_\alpha{}^T$ are the mobility and thermal diffusion ratio of species α, respectively, and κ_e is the electron thermal conductivity. These transport coefficients are defined by the relations

$$\mu_\alpha\equiv\frac{2e}{3n_\alpha{}^2m_\alpha}\int f_\alpha{}^0u_\alpha A_\alpha(u_\alpha)d^3v_\alpha, \tag{26a}$$

$$k_\alpha{}^T\equiv\frac{2e}{3n_\alpha{}^2m_\alpha\mu_\alpha}\int f_\alpha{}^0\mu_\alpha B_\alpha(u_\alpha)d^3v_\alpha, \tag{26b}$$

$$\kappa_e\equiv\frac{\mu_en_ek^2T_e}{e}\left[\frac{2e}{3n_e{}^2m_e\mu_e}\int f_e{}^0u_e{}^3B_e(u_e)d^3v_e-k_e{}^T(\tfrac{5}{2}+k_e{}^T)\right]. \tag{26c}$$

In deriving Eqs. (24)–(26) use is made of the relation

$$\int f_\alpha{}^0(u_\alpha{}^2-\tfrac{5}{2})\mathbf{A}_\alpha\cdot\mathbf{u}_\alpha d^3v_\alpha\equiv\int f_\alpha{}^0\mathbf{B}_\alpha\cdot\mathbf{u}_\alpha d^3v_\alpha, \tag{27}$$

which is readily established from Eqs. (17)–(20) and the symmetry property [Eq. (15)] of the I_α and $I_{\alpha\beta}$ operators.

The last terms in Eqs. (24) are the "thermal diffusion" terms of Chapman and Cowling[3] and other authors. The first term on the right-hand side of Eq. (25) represents enthalpy transport, the second term arises from the "diffusion thermoeffect," an energy transport mechanism which is related to the thermal diffusion mechanism in Eqs. (24a), and the third term accounts for thermal conduction. The definitions of the transport coefficients μ_α, $k_\alpha{}^T$, and κ_e are consistent with the usual definitions of these quantities. It should be noted that the thermal diffusion ratio appears as a transport coefficient in both Eqs. (24a) and (25). This fact is a direct consequence of Eq. (27) and is in agreement with a general reciprocity theorem relating to the thermodynamics of irreversible processes.[6]

In the case of a Lorentz plasma Eqs. (17) and (18) may be solved exactly for the $A_e(u_e)$ and $B_e(u_e)$ functions which may in turn be used in Eqs. (26) to evaluate the electron transport coefficients (see Sec. 5). To evaluate the ion transport coefficients, as well as the electron transport coefficients, in a general three-component plasma, it is necessary to resort to the Sonine expansions. Substitution of these expansions into Eqs. (26) yields the relations:

$$\mu_\alpha=\frac{ea_\alpha{}^{(0)}}{n_\alpha m_\alpha};\quad k_\alpha{}^T=\frac{b_\alpha{}^{(0)}}{a_\alpha{}^{(0)}};\qquad \kappa_e=-\frac{5}{2}\frac{k^2T_e}{m_e}[b_e{}^{(1)}+\tfrac{2}{5}k_e{}^Tb_e{}^{(0)}]. \tag{28}$$

Note that the plasma transport coefficients depend explicitly only upon a finite (and indeed small) number of Sonine expansion coefficients regardless of how many terms are retained in the Sonine expansion Eqs. (21). This convenient result is a consequence of the use of the Sonine polynomials of order $\frac{3}{2}$ in Eqs. (21); had other orthogonal functions been used, Eqs. (28) would contain terms corresponding to an infinite number of expansion coefficients.

The ion force component $\mathbf{R}_{ie}$, arising from the collisional transfer of directed electron momentum to the ions, can also be written in terms of the Sonine expansion coefficients:

$$\mathbf{R}_{ie}=-[\sum_{n=0}^{\infty}(\nu_{ei}{}^{(n)}/n_e)a_e{}^{(n)}](\nabla_rp_e+en_e\mathbf{E})-[\sum_{n=0}^{\infty}(\nu_{ei}{}^{(n)}/n_e)b_e{}^{(n)}]n_ek\nabla_rT_e, \tag{29}$$

[6] L. Onsager, Phys. Rev. 38, 2265 (1931).

where

$$\nu_{ei}^{(n)} = (2/3n_e) \int f_e^0 u_e^2 S_n^{\frac{3}{2}}(u_e^2)\nu_{ei}(v_e) d^3v_e.$$

4.3. Inelastic Collision Terms

The inelastic collision terms (S_e^{in}, S_i^{in}, and Q_e^{in}) in Eqs. (23) may be expressed in terms of the plasma densities and temperatures once the dominant inelastic collisional processes are specified. Since these processes vary greatly according to the plasma under consideratiin, the evaluation of the inelastic collision terms must be related to specific applications of the plasma transport differential equations. Such applications are not discussed in this paper. It should be noted, however, that the inelastic collision terms may depend strongly upon the high-energy tail of the electron distribution function. If the plasma conditions are such that the high-energy electrons are nearly Maxwellian, then the electron distribution functions of Sec. 3 are applicable. If this condition is not fulfilled, appropriate consideration must be given to the depletion of the tail of the electron velocity distribution.

5. EVALUATION OF TRANSPORT COEFFICIENTS

5.1. Transport Coefficients for Lorentz Plasmas

In the case of a Lorentz plasma, Eqs. (17)–(18) may be solved exactly. Hence the Lorentz plasma provides a convenient reference case for comparisons. More specifically, for a Lorentz plasma consisting of electrons and heavy particles of species β only, the electron perturbation function and transport coefficients are given by the relations:

$$\phi_e(\mathbf{r},\mathbf{v}_e) = -\lambda_{e\beta}(v_e)\left[\frac{\nabla_r p_e}{p_e} + \frac{e\mathbf{E}}{kT_e} + (u_e^2 - \tfrac{5}{2})\frac{\nabla_r T_e}{T_e}\right]\frac{\mathbf{u}_e}{u_e}, \quad (30)$$

$$\mu_e = \frac{e}{p_e}\int \frac{v_e^2}{3\nu_{e\beta}(v_e)} f_e^0 d^3v_e, \quad (31a)$$

$$k_e^T = \frac{e}{\mu_e p_e}\int \frac{v_e^2}{3\nu_{e\beta}(v_e)}\left(\frac{m_e v_e^2}{2kT_e} - \frac{5}{2}\right) f_e^0 d^3v_e, \quad (31b)$$

$$\kappa_e = \frac{\mu_e k p_e}{e}\left[\frac{e}{\mu_e p_e}\int \frac{v_e^2}{3\nu_{e\beta}(v_e)} \times \left(\frac{m_e v_e^2}{2kT_e}\right)^2 f_e^0 d^3v_e - (\tfrac{5}{2} + k_e^T)^2\right], \quad (31c)$$

where $\lambda_{e\beta}(v_e) \equiv v_e/\nu_{e\beta}(v_e)$ is the electron free path. Note that the perturbation $\phi_e(\mathbf{r},\mathbf{v}_e)$ is small compared to unity when the fractional changes in the electron pressure and temperature and in the plasma potential are small over one free path.

TABLE I. Transport coefficients for Lorentz plasmas.

Heavy-particle species / Collision law	Neutrals (constant ν_{eo})	Neutrals (constant λ_{eo})	Ions (C)
μ_e	$\dfrac{e}{m_e\nu_{eo}}$	$\dfrac{e}{m_e\hat{\nu}_{eo}}$	$\dfrac{e}{m_e\hat{\nu}_{ei}}$
k_e^T	0	$-\frac{1}{2}$	$-\frac{3}{2}$
κ_e	$\dfrac{5n_ek^2T_e}{2m_e\nu_{eo}}$	$\dfrac{2n_ek^2T_e}{m_e\hat{\nu}_{eo}}$	$\dfrac{4n_ek^2T_e}{m_e\hat{\nu}_{ei}}$

Shown in Table I are values of the transport coefficients obtained from Eqs. (31) for three Lorentz plasmas of special interest: (a) the heavy particles are neutrals and $\nu_{eo}(v_e) = \text{constant}$, (b) the heavy particles are neutrals and $\lambda_{eo}(v_e) = \text{constant}$, (c) the heavy particles are ions. Results for cases (b) and (c) are presented in terms of effective electron–neutral and electron–ion collision frequencies defined by

$$\hat{\nu}_{eo} \equiv \frac{3\pi^{\frac{1}{2}} n_0 \sigma_{eo}(kT_e)^{\frac{1}{2}}}{2^{\frac{3}{2}} m_e^{\frac{1}{2}}}; \quad \hat{\nu}_{ei} \equiv \frac{2^{\frac{1}{2}} n_i e^4 \ln\Lambda_e}{128\pi^{\frac{1}{2}}\epsilon_0^2 m_e^{\frac{1}{2}}(kT_e)^{\frac{3}{2}}},$$

where σ_{eo} is the electron–neutral hard-sphere cross section and $\ln\Lambda_e$ is the Coulomb logarithm for electrons. Note that a Lorentz plasma in which the heavy particles are ions cannot be achieved experimentally because of the existence of charge neutrality in laboratory plasmas. The transport coefficients in Table I are in agreement with generally accepted values.

Truncation errors associated with terminating the Sonine expansions after N terms can be estimated by using the Sonine expansion technique to evaluate the transport coefficients for a Lorentz plasma, and comparing the results with the exact values given in Table I. This comparison for a Lorentz plasma with constant λ_{eo} reveals (Table II) that the retention of three terms in each of the Sonine expansions is sufficient to yield all three transport coefficients correct to within 10%.

5.2. Transport Coefficients for a Three-Component Plasma

The formalism of Secs. 2, 3, and 4 may be used to evaluate the transport coefficients and $\mathbf{R}_{ie}$ in three-component plasmas. For example, consider a three-

TABLE II. Transport coefficients for a Lorentz plasma with constant λ_{eo}.

Number of terms in Sonine expansion	$\mu_e/(\mu_e)_{\text{exact}}$	$k_e^T/(k_e^T)_{\text{exact}}$	$\kappa_e/(\kappa_e)_{\text{exact}}$
$N=1$	0.88	0	0
$N=2$	0.95	0.77	0.85
$N=3$	0.98	0.90	0.93

component plasma in which: (a) the charged particle elastic collisions may be treated as Coulomb interactions, (b) the charged particle–neutral particle elastic collisions may be treated as hard-sphere interactions, and (c) $n_e \simeq n_i$. With the collision laws so specified, the matrices, $[\delta_\alpha^{(m,n)}]$; $\alpha = e, i$, of Eqs. (22) may be readily evaluated. Indeed, if three terms are retained in each of the Sonine expansions:

$$[\delta_e^{(m,n)}] = \frac{\nu_{eo}}{n_e}\begin{bmatrix} 1.70+5.09\tau_e & -0.85+7.64\tau_e & -0.21+9.55\tau_e \\ -1.85+7.64\tau_e & 5.52+23.8\tau_e & -2.44+27.4\tau_e \\ -0.21+9.55\tau_e & -2.44+27.4\tau_e & 11.5+54.7\tau_e \end{bmatrix}, \tag{32a}$$

$$[\delta_i^{(m,n)}] = \frac{\nu_{io}}{n_i}\begin{bmatrix} 1.70 & -0.42 & -0.05 \\ -0.42 & 6.25+10.2\tau_i & -1.87+7.64\tau_i \\ -0.05 & -1.87+7.64\tau_i & 13.8+28.6\tau_i \end{bmatrix}, \tag{32b}$$

where effective ion–neutral and ion–ion collision frequencies are defined by

$$\hat{\nu}_{io} \equiv \frac{3\pi^{\frac{1}{2}} n_o \sigma_{io}(kT_i)^{\frac{1}{2}}}{4m_i^{\frac{1}{2}}}; \quad \hat{\nu}_{ii} \equiv \frac{n_i e^4 \ln\Lambda_i}{128\pi^{\frac{1}{2}}\epsilon_0^2 m_i^{\frac{1}{2}}(kT_i)^{\frac{3}{2}}},$$

σ_{io} is the ion–neutral hard-sphere cross section, $\ln\Lambda_i$ is the Coulomb logarithm for ions, $\tau_e \equiv \hat{\nu}_{ei}/\hat{\nu}_{eo}$, and $\tau_i \equiv \hat{\nu}_{ii}/\hat{\nu}_{io}$. Thus Eqs. (22) yield $a_\alpha^{(n)}$ and $b_\alpha^{(n)}$, which in turn can be used in Eqs. (28)–(29) to calculate the transport coefficients and $\mathbf{R}_{ie}$. The results of this calculation are presented below.

Electron Transport Coefficients: The second column of Table III gives the values of the electron transport coefficients in the limit $\tau_e = 0$. They are denoted by the superscript "0" and are within 10% of the exact values given in Table I. (see also Table II).

The third column of Table III gives the values of the electron transport coefficients in the limit $\tau_e = \infty$. They are denoted by the superscript "∞" and are in excellent agreement with results presented in Refs. 4 and 7. Comparison of these transport coefficients with those given in Table I for an electron–ion Lorentz plasma reveals that the effect of electron–electron collisions in a fully ionized plasma is to reduce each of the electron transport coefficients to 0.25–0.50 of the value obtained in the absence of electron–electron collisions.

TABLE III. Electron transport coefficients.

	Limit $\tau_e \ll 1$	Limit $\tau_e \gg 1$	General expression
μ_e	$\mu_e^0 = \dfrac{0.98e}{m_e\hat{\nu}_{eo}}$	$\mu_e^\infty = \dfrac{0.58e}{m_e\hat{\nu}_{ei}}$	$\mu_e = \dfrac{\mu_e^0\mu_e^\infty}{\mu_e^0+\mu_e^\infty} h_{\mu e}(\tau_e)$
k_e^T	$(k_e^T)^0 = -0.45$	$(k_e^T)^\infty = 0.71$	$k_e^T = k_e^T(\tau_e)$
κ_e	$\kappa_e^0 = \dfrac{1.88 n_e k^2 T_e}{m_e\hat{\nu}_{eo}}$	$\kappa_e^\infty = \dfrac{0.93 n_e k^2 T_e}{m_e\hat{\nu}_{ei}}$	$\kappa_e = \dfrac{\kappa_e^0\kappa_e^\infty}{\kappa_e^0+\kappa_e^\infty} h_{\kappa e}(\tau_e)$

[7] M. V. Samokhin, Soviet Phys.—Tech. Phys. 8, 498 (1963).

The fourth column of Table III gives expressions for the electron transport coefficients for $0 \leqslant \tau_e \leqslant \infty$. The functions $h_{\mu e}(\tau_e)$, $k_e^T(\tau_e)$, and $h_{\kappa e}(\tau_e)$ are

$$h_{\mu e}(\tau_e) = \frac{1.00+14.1\tau_e+30.6\tau_e^2+16.3\tau_e^3}{1.00+21.1\tau_e+37.4\tau_e^2+16.3\tau_e^3},$$

$$k_e^T(\tau_e) = \frac{-0.45+3.07\tau_e+6.83\tau_e^2}{1.00+12.4\tau_e+9.83\tau_e^2},$$

$$h_{\kappa e}(\tau_e) = \frac{1.00+25.5\tau_e+176\tau_e^2+454\tau_e^3+471\tau_e^4+158\tau_e^5}{1.00+31.5\tau_e+284\tau_e^2+662\tau_e^3+562\tau_e^4+158\tau_e^5}.$$

The functions $h_{\mu e}(\tau_e)$ and $h_{\kappa e}(\tau_e)$ are plotted in Fig. 1 and the electron thermal diffusion ratio $k_e^T(\tau_e)$ is plotted in Fig. 2. Note that $h_{\mu e}(\tau_e)$ is close to unity for all values of $\tau_e (0 \leqslant \tau_e \leqslant \infty)$. This implies that, to a good approximation, the contributions to the electron mobility from electron–neutral collisions and from electron–charged-particle collisions may be added in parallel. Similar conclusions apply to the contributions to the thermal conductivity of the electrons. These conclusions are, of course, restricted to the collision laws under

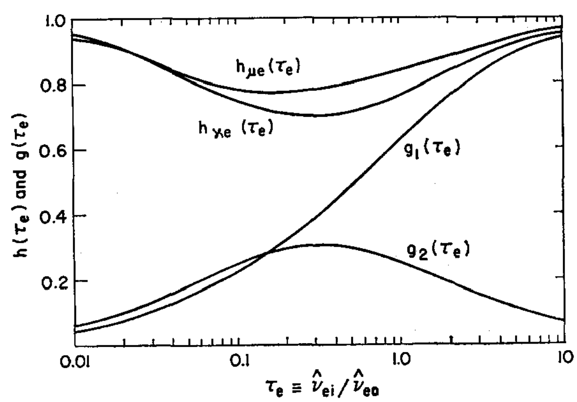

FIG. 1. Plots of the $h(\tau_e)$ and $g(\tau_e)$ functions vs τ_e when three terms are retained in the Sonine polynomial expansions.

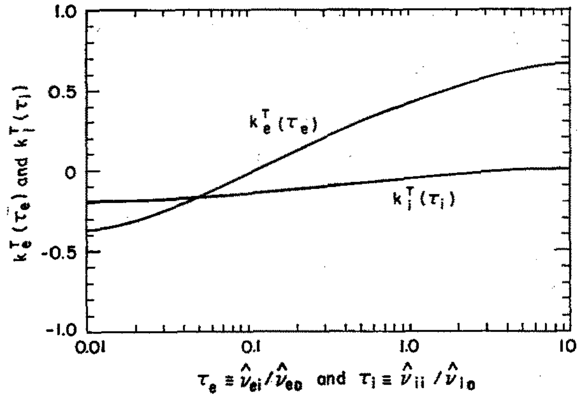

FIG. 2. Plots of the electron and ion thermal diffusion ratios vs τ_e and τ_i, respectively, when three terms are retained in the Sonine polynomial expansions.

consideration and do not necessarily apply to other collision laws.

Ion Transport Coefficients: The ion transport coefficients are given in Table IV in a form analogous to that of the electrons (Table III). The functions $h_{\mu i}(\tau_i)$ and $k_i^T(\tau_i)$ are

$$h_{\mu i}(\tau_i)=\frac{1.00+4.20\tau_i+2.86\tau_i^2}{1.00+4.24\tau_i+2.91\tau_i^2},$$

$$k_i^T(\tau_i)=\frac{0.18+0.35\tau}{1.00+4.20\tau_i+2.86\tau_i^2}.$$

Note that the ion mobility is practically independent of τ_i since the function $h_{\mu i}(\tau_i)$ is within 2% of unity in the entire range $0\leqslant\tau_i\leqslant\infty$. The ion thermal diffusion ratio $k_i^T(\tau_i)$ is plotted in Fig. 2.

Ion Force Component $\mathbf{R}_{ie}$: The ion force component $\mathbf{R}_{ie}$ is found to be

$$\mathbf{R}_{ie}=-g_1(\tau_e)(\nabla_r p_e+en_e\mathbf{E})+g_2(\tau_e)n_e k\nabla_r T_e, \quad (33)$$

where

$$g_1(\tau_e)\equiv\frac{4.56\tau_e+25.2\tau_e^2+16.3\tau_e^3}{1.00+19.1\tau_e+37.4\tau_e^2+16.3\tau_e^3},$$

$$g_2(\tau_e)\equiv\frac{6.88\tau_e+11.7\tau_e^2}{1.00+19.1\tau_e+37.4\tau_e^2+16.3\tau_e^3}.$$

The functions $g_1(\tau_e)$ and $g_2(\tau_e)$ are plotted in Fig. 1.

TABLE IV. Ion transport coefficients.

	Limit $\tau_i\ll1$	Limit $\tau_i\gg1$	General expression
μ_i	$\mu_i^0=\dfrac{0.90e}{m_i\hat{\nu}_{io}}$	$\mu_i^\infty=\dfrac{0.89e}{m_i\hat{\nu}_{io}}$	$\mu_i=\mu_i^0h_{\mu i}(\tau_i)$
k_i^T	$(k_i^T)^0=-0.18$	$(k_i^T)^\infty=0$	$k_i^T=k_i^T(\tau_i)$

Note that in the limit $\tau_e=\infty$, $g_1(\tau_e)=1$, $g_2(\tau_e)=0$, and $\mathbf{R}_{ie}=-(\nabla_r p_e+en_e\mathbf{E})$. This relation may also be obtained directly from Eqs. (4b) if $\nabla_r\cdot\bar{\pi}_\alpha$, $\mathbf{R}_\alpha^{in}$, and $\mathbf{R}_{\alpha o}$ are neglected.

SUMMARY AND CONCLUSIONS

A novel set of plasma transport differential equations suitable for the analysis of transport phenomena in low-energy three-component, two-temperature plasmas is derived. The equations are obtained through a two-step procedure which involves: (a) solving a set of linearized Boltzmann equations for the charged-particle distribution functions in near-Maxwellian plasmas and (b) using these distribution functions to reduce the single-species, charged-particle hydrodynamic equations to a set of differential equations involving the plasma densities, temperatures, and electric field, and a set of transport coefficients which depend upon the prevailing elastic collision laws.

The form of the derived transport equations is independent of the prevailing collision laws; these laws affect only the values of the transport coefficients. The transport coefficients may be computed numerically, if the collision laws are known, or inferred from experimental measurements.

Transport coefficients for Lorentz plasmas can be computed exactly. Values obtained herein for Lorentz plasmas of interest are in agreement with accepted values.

The transport coefficients for the particular three-component plasma considered in Sec. 5 can be readily interpreted physically. Indeed, the general expression for the electron mobility μ_e is given to a good approximation by adding the mobilities μ_e^0 and μ_0^∞ in parallel. The same approximation is true for the electron thermal conductivity. The ion mobility, however, is practically independent of the ion collision frequency since ion–ion collisions do not alter the net ion momentum.

The error in the transport coefficients due to the truncation of the Sonine expansions after three terms is less than 10%. This follows from the comparison of the computed electron transport coefficients for the limit $\tau_e\ll1$ with corresponding exact values for these coefficients.

The ion force component $\mathbf{R}_{ie}$, arising from the collisional transfer of directed electron momentum to the ions, becomes important in the expression for the ion current when $\tau_e\geqslant O(1)$. This term has been neglected in previous studies.

Electron–electron collisions are as important as electron–ion collisions in determining the electron transport coefficients. This follows from the comparison of the computed electron transport coefficients for $\tau_e\gg1$ with exact values for these coefficients in the absence of electron–electron collisions.