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from fast boosters must drift to 120– 140 km to deploy reentry vehicles and decoys deceptively,^{2.4} so if irradiation started at 110 km and ended at 150 km, the 0.25-sec average kill time would allow each neutral particle beam to negate about 60 buses, which is appropriate for nominal constellations. The altitudes did not bias our assessment of neutral particle beam effectiveness, which should continue into the long term.

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Temperature: A Matter of Degrees

8/88

Is temperature defined for systems with one degree of freedom? Many believe that the answer to the question is no (such as Herman Feshbach in PHYSICS TODAY, November 1987, page 9). I would like to present a different viewpoint.

The concept of the temperature of a system is well defined in theory, and very demanding in application. The *thermodynamic temperature* is

$$T = (\partial E / \partial S)_{n,a} \tag{1}$$

without any restrictions on n and a, where n denotes the values of the amounts (which are equivalent to the number of degrees of freedom) of the constituents, and a the values of the constraints.

This definition applies to states for which the energy *E* depends only on the entropy *S* and on *n* and *a*. The only states for which E = E(S,n,a) are the thermodynamic equilibrium

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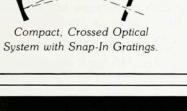
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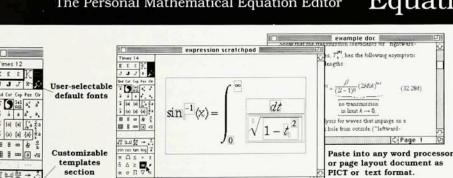
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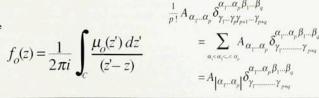
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states. Each of these has the smallest (or largest) energy of all the states with the same values of S, n and a, or, alternatively, the largest entropy of all the states with the same values of E, n and a. (Spin systems have energies that are bounded both from below and from above. For each set of values S, n and a, the state with the smallest energy corresponds to a positive temperature, and the state with the largest to a negative temperature.) For other states, E depends on more variables, and equation 1 is inappropriate.

Of course, one might consider states with more independent variables and define a *T* by either a similar derivative or some other procedure. However, such a definition would be useless for the thermodynamics of equilibrium. For a combination of two systems A and B to be in a thermodynamic equilibrium state, the entropy of that state must be the largest of all the states with the same values of $E^{\rm A} + E^{\rm B}$, $n^{\rm A} + n^{\rm B}$ and $a^{\rm A} + a^{\rm B}$. A necessary condition for this to be true is that

 $\left[(\partial E/\partial S)_{n,a} \right]^{\mathrm{A}} = \left[(\partial E/\partial S)_{n,a} \right]^{\mathrm{B}}$

or

$$T^{\rm A} = T^{\rm B} \tag{2}$$

that is, that A and B be in temperature equality. Because only the thermodynamic temperature satisfies equation 2, any definition of T different from equation 1 would be irrelevant to the most important and unique operational meaning we assign to temperature measurements.

In probabilistic interpretations, the question arises of whether temperature is restricted to systems with particular values of either the amounts or the constraints, or both. Since the time of Josiah Willard Gibbs,¹ it has been recognized that no such restriction is necessary. Temperature is defined for systems with one degree of freedom and any values of the constraints. To see this, consider a system with a Hamiltonian operator \hat{H} , and the energy eigenvalue problem

$$\widehat{\mathsf{H}}u_i \stackrel{\cdot}{=} \epsilon_i u_i, \quad i = 1, 2, \ldots$$

where the spectrum ϵ_i is discrete, and each ϵ_i is a function of n and a. No restrictions are imposed on n and abecause none are required by the eigenvalue problem; that is, the system may be one spin with two eigenstates, one harmonic oscillator or many particles in a box (large or small) with many eigenstates.

Regardless of the system, an equi-

librium state has values of energy and entropy

$$E = \sum_{i} x_{i} \epsilon_{i}$$
$$S = -k \sum_{i} x_{i} \ln x_{i}$$

where

$$x_i = \frac{\exp(-\epsilon_i/kT)}{\sum_i \exp(-\epsilon_i/kT)}$$

and is found² by maximizing *S* subject to fixed values of *E*, *n* and *a*. Here, the value of *T* is determined by *E*, and satisfies the relation

$$T = (\partial E/\partial S)_{\epsilon_i} = (\partial E/\partial S)_{n,a}$$

That is, T is the temperature, where the subscript ϵ_i denotes that all ϵ_i are kept fixed. [For this elegant and concise derivation of the canonical distribution, we need not invoke either a heat bath or interactions between subsystems of the overall system. All we need is to admit the existence of quantum theoretic probability distributions associated with a homogeneous ensemble, even though its density operator may have the form traditionally associated with inhomogeneous quantal ensembles (mixtures).³] Because no restrictions are imposed on n and a, temperature is defined for thermodynamic equilibrium states of systems with any number of degrees of freedom, including 1, any number of energy eigenvalues greater than or equal to 2, and any values of constraints.

Concerns about the applicability of the concept of temperature are sometimes expressed in terms of fluctuations. (This is how Feshbach expressed his concerns.) Some of these concerns, however, are misplaced. One meaning of the term "fluctuation" relates to the variance of a probability distribution associated with measurement results of an observable, such as the variances entering the Heisenberg uncertainty principle. Regardless of the magnitude of this fluctuation, the (expectation) value of the corresponding observable is well defined, and no restriction is imposed on its validity. So the fluctuation of temperature discussed by Feshbach, on the basis of which he concludes that "for small systems thermodynamics is not always quantitatively valid," cannot be variancelike. Besides, temperature is not a quantum observable.

The other meaning relates to changes over time. Such fluctuations imply that a system is not in equilibrium. But then the concept of temperature does not apply, not because the number of degrees of freedom is small, but because the state is not equilibrium.

I am thankful to Feshbach, Gian Paolo Beretta and Marvin Miller for helpful discussions.

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FESHBACH REPLIES: It is always possible to devise a precise definition of temperature. Such precision does not preclude the existence of fluctuations. In my Reference Frame column I called attention to the dependence of these fluctuations for isolated systems on the number of particles the system contains.

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Cluster Credit: Who Seconded the Motion?

I enjoyed reading Per Andersen's news story (October 1987, page 17) on recent work on nonuniformities in the Hubble flow. However, the first study to indicate large-scale motion was not that of Vera Rubin, Norbert Thonnard, W. Kent Ford Jr and Morton Roberts¹; in 1973, I found² that the Virgo cluster was receding faster than its expected Hubble velocity by several hundred kilometers per second. I used two independent methods of gauging relative cluster distances, the radius-magnitude relation and the m^* method of George Abell.³ Five years later, with improved data for the m^* method, and the first application of the microwave background correction⁴ to cluster redshifts, I refined⁵ the Virgo peculiar velocity to 658 ± 96 km/sec. There was also preliminary evidence for a peculiar velocity for cluster Abell 1367 of 683 km/sec. Rubin and her colleagues interpreted their results as showing motion of our Galaxy, not motion of