

NUCLEAR REACTOR DYNAMICS

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### 1. INTRODUCTION

The subject of the time behavior of the neutron population or power of a nuclear reactor has been treated by many authors and a number of analytical models have been developed for this purpose. All models are based on universally accepted conservation principles such as conservation of particles, energy etc. In particular, the time dependent neutron balance equation is always an analytical statement of the equality.

$$\left[ \begin{array}{c} \text{Rate of Change} \\ \text{of Neutron} \\ \text{Population} \end{array} \right] = \left[ \begin{array}{c} \text{Rate} \\ \text{of} \\ \text{Production} \end{array} \right] - \left[ \begin{array}{c} \text{Rate} \\ \text{of} \\ \text{Destruction} \end{array} \right] \quad (1)$$

The difference between models lies in the theoretical concepts that are used to express quantitatively the various physical processes that affect the rates of production and destruction.

The most basic method of analytically implementing equality (1) is by means of the concepts of transport theory. This theory allows a precise and detailed representation of all conceivable interactions that neutrons undergo in a reactor with a minimum number of assumptions. Of course, the equations that result are very difficult to handle because of their generality. However, they can be reduced to more tractable forms by appropriate mathematical manipulations and convenient definitions of directly or indirectly measurable parameters. One of these tractable forms is the space inde-

pendent conventional form of reactor kinetics which has proved a very productive tool in reactor dynamics studies.

The purpose of this section is to derive the conventional form of reactor kinetics equations starting from the basic notions of transport theory. The value of this approach is that the exact meaning of such convenient parameters as reactivity, neutron lifetime etc. can be readily indicated and that all the assumptions and limitations inherent in the conventional form of reactor kinetics can be established.

## 2. REACTOR KINETICS & TRANSPORT THEORY

Consider a reactor in which the fuel is stationary. For simplicity and without any loss of generality, assume that there is only one species of fissionable material.

The time dependent balance equations for the neutron population and the delayed neutron precursor concentrations are (for terminology see nomenclature)

$$\begin{aligned} \frac{\partial}{\partial t} N(\vec{r}, E, \Omega, t) = & \\ & \frac{1}{4\pi} \rho_0(E) \int_{\Omega'} d\Omega' \int_0^{\infty} dE' v(E') (1-\beta) v' \sum_f(\vec{r}, E', t) N(\vec{r}, E', \Omega', t) + \\ & + \int_{\Omega} d\Omega' \int_0^{\infty} dE' v' \sum_s(\vec{r}, E' \rightarrow E, \Omega' \rightarrow \Omega, t) N(\vec{r}, E', \Omega', t) \\ & + \frac{1}{4\pi} \sum_{i=1}^M \lambda_i \rho_i(E) C_i(\vec{r}, t) + S(\vec{r}, E, \Omega, t) - \\ & - \Omega v \cdot \text{grad} N(\vec{r}, E, \Omega, t) - v \sum_t(\vec{r}, E, t) N(\vec{r}, E, \Omega, t) \quad (2) \end{aligned}$$

$$\frac{\partial C_i(\bar{r}, t)}{\partial t} = \int_{\Omega'} d\Omega' \int_0^{\infty} dE' v(E') \beta_i v' \Sigma_f(\bar{r}, E', t) N(\bar{r}, E', \Omega', t) - \lambda_i C_i(\bar{r}, t) \quad i=1, 2, \dots, m \quad (3)$$

The meaning of Eqs. (2) and (3) may be understood as follows:

Consider an elementary volume  $\Delta V$  centered around the arbitrary point  $\bar{r}$  of the reactor (Figure 1). Define the neutron density  $N(\bar{r}, E, \Omega, t)$ . This density is the number of neutrons per unit volume, per unit energy, per unit solid angle, that are at position  $\bar{r}$ , have an energy  $E$  and move in the direction  $\Omega$  at time  $t$ .

The rate of change of the neutrons in the elementary volume  $\Delta V \Delta \Omega$  of the phase space  $(\bar{r}, \Omega)$  per unit energy interval at the energy level  $E$  is:

$$\Delta V \Delta \Omega \frac{\partial}{\partial t} N(\bar{r}, E, \Omega, t) \quad (4)$$

Neutrons are created in this elementary phase space volume through the following production mechanisms:

- a. Prompt fissions
- b. Elastic and inelastic scattering collisions
- c. Decay of delayed neutron precursors
- d. External neutron sources

Thus, the rate of production in  $\Delta V \Delta \Omega$  can be written as

$$\left[ \frac{1}{4\pi} v_0(E) \int_{\Omega'} d\Omega' \int_0^\infty dE' v(E') (1-\beta) v' \sum_f(\bar{r}, E', t) N(\bar{r}, E', \Omega, t) + \right.$$

(prompt fissions)

$$\left. + \int_{\Omega'} d\Omega' \int_0^\infty dE' \sum_s(\bar{r}, E' \rightarrow E, \Omega' \rightarrow \Omega, t) N(\bar{r}, E', \Omega', t) + \right.$$

(scattering collisions)

$$\left. + \frac{1}{4\pi} \sum_{i=1}^M \lambda_i f_i(E) C_i(\bar{r}, t) + S(\bar{r}, E, \Omega, t) \right] \Delta V \Delta \Omega \tag{5}$$

(delayed neutrons)      (source)

On the other hand, neutrons are lost from the elementary phase space volume through the following destruction mechanisms:

- a. Leakage
- b. Absorption\*

Thus, the destruction rate can be written as

$$\left[ -\nabla \cdot \text{grad} N(\bar{r}, E, \Omega, t) + v \sum_t(\bar{r}, E, t) N(\bar{r}, E, \Omega, t) \right] \Delta V \Delta \Omega \tag{6}$$

(Leakage)                      (Total absorption)

It is evident that the combination of Eqs. (4-6) as indicated by the principle of conservation of particles (Eq. (1)) yields Eq. (2).

A similar procedure can be used to verify Eq. (3) for the delayed neutron concentrations.

Notice that in writing Eqs. (2-3) it is assumed that both fission neutrons and delayed neutrons are emitted isotropically.

\*Including scattering-out collisions

In addition, note that a time dependence of the various cross sections is introduced to account for the effects of the non-nuclear processes and external disturbances on the neutron balance.

Eqs. (2) and (3) are basic and general as already indicated. Their generality, however, renders them extremely difficult to use in this form for analytical studies. Consequently, additional simplifications and approximations are necessary to reduce them to a more tractable form. The reduction can be achieved by a variety of mathematical manipulations. Which manipulation is appropriate is a matter of expediency depending on the reactor type and the particular aspect of reactor dynamics that is under investigation. For example, series expansions of the neutron density in terms of reactor period eigenfunctions, reactivity eigenfunctions, or other sets of eigenfunctions reduce Eqs. (2) and (3) to an infinite set of ordinary differential equations with respect to time, which may be more readily amenable to computations.

Another approach to the problem of simplification of Eqs. (2) and (3) is to proceed as in the following section.

### 3. REACTOR KINETICS - CONVENTIONAL FORM

The major difficulty with the kinetics equations derived from transport theory is their implicit or explicit dependence on the neutron energy, direction of motion and position. It is obvious that they would be much easier to handle if they

were dependent only on time. To this end it is practical to integrate (average) the energy, direction and position dependencies out of the transport theory equations. The result is the conventional form of reactor kinetics.

The question now is how to perform the averaging. If the energy and phase space dependencies were completely known (viz. the neutron and delayed neutron precursor densities were known!) then equations (2) and (3) could be readily integrated. Of course, this is not the case and one has to approximate these dependencies and then integrate. This is called linear averaging and it implies that the error inherent in the approximation is directly or linearly reflected on the ordinary time dependent differential equations resulting from the integration.

The error in the integrated equations can be reduced if the unknown functions in Eqs. (2) and (3) are first multiplied by some weighing factors and then integrated. In fact, it is shown that the averaging is exact if the weighing factors are selected from the exact solutions of the adjoint problem. Of course, the solutions of the adjoint problem are not known either, and one has to use approximate arbitrary weighing factors as well as energy, space and direction dependencies in Eqs. (2) and (3). This is called bilinear averaging and it implies that the error in the resulting differential equations is of second order with respect to the errors made in the approximations of the energy and phase space dependencies and the weighing factors.

In view of these remarks, consider the source free adjoint equation, corresponding to some critical condition of the reactor.

$$\int_{\Omega} d\Omega' \int_0^{\infty} dE' \left[ v' \sum_{s_0} (\bar{r}, E' \rightarrow E, \Omega' / \Omega) + \right. \\ \left. + \frac{1}{4\pi} f_t(E) v(E') v' \sum_{f_0} (\bar{r}, E') \right] N_0^*(\bar{r}, E', \Omega') + \\ + \Omega v \cdot \text{grad} N_0^*(\bar{r}, E, \Omega) - v \sum_t (\bar{r}, E) N_0^*(\bar{r}, E, \Omega) = 0 \quad (7)$$

$$f_t(E) = f_0(E) (1 - \beta) + \sum_{i=1}^M f_i(E) \beta_i \quad (8)$$

Assume that the adjoint density  $N_0^*(\bar{r}, E, \Omega)$  of the critical reactor is known. In addition, write the time dependent neutron density in the form

$$N'(\bar{r}, E, \Omega, t) = P(t) N_0(\bar{r}, E, \Omega, t) \quad (9)$$

with the additional requirement that

$$\frac{\partial}{\partial t} \int_V d\bar{r} \int_{\Omega} d\Omega \int_0^{\infty} dE N_0^*(\bar{r}, E, \Omega) N_0(\bar{r}, E, \Omega, t) = 0 \quad (10)$$

Note that Eqs. (9) and (10) are mere definitions and do not involve any assumptions. A special case where they are readily satisfied is when the time and all other coordinates are separable.

Next, replace Eq. (9) into Eqs. (2) and (3), multiply both sides of Eqs. (2) and (3) and (7) by  $N_0^*(\bar{r}, E, \Omega)$  and  $N_0(\bar{r}, E, \Omega, t)$  respectively and integrate over the entire volume of the reactor, over all energies and over all solid angles. Thus find that Eqs. (2) and (3) reduce to the form



$$\frac{d}{dt}P(t) = \frac{P-\bar{P}}{\Lambda} P(t) + \sum_{i=1}^m \lambda_i c_i(t) + Q(t) \quad (11)$$

$$\frac{d}{dt}c_i(t) = \frac{\bar{P}}{\Lambda} P(t) - \lambda_i c_i(t) \quad i=1,2,\dots,m \quad (12)$$

where, if

$$\int_u du \dots = \int_V d\vec{r} \int_0^\infty dE \int_\Omega d\Omega \dots$$

$$\begin{aligned} \rho = \rho(t) = & \frac{1}{V} \int_u du \left[ N_0^*(\vec{r}, E, \Omega) \frac{f_1(E)}{4\pi} \int_{\Omega'} d\Omega' \int_0^\infty dE' v(E') v' \delta \sum_r N_0(\vec{r}, E', \Omega', t) + \right. \\ & \left. + \int_{\Omega'} d\Omega' \int_0^\infty dE' v' \delta \sum_s N_0(\vec{r}, E', \Omega', t) - v \delta \sum_t N_0(\vec{r}, E', \Omega', t) \right. \\ & \left. - N_0^*(\vec{r}, E, \Omega) \Omega v \cdot \text{grad} N_0(\vec{r}, E, \Omega, t) - \right. \\ & \left. - N_0(\vec{r}, E, \Omega, t) \Omega v \cdot \text{grad} N_0^*(\vec{r}, E, \Omega) \right] \quad (13) \end{aligned}$$

$\delta \sum_i$  represents the difference between the time dependent and time independent cross sections.

$$P = P(t) = \int_u du N_0^*(\vec{r}, E, \Omega) \sum_{i=1}^m \frac{f_i(E)}{4\pi} \int_{\Omega'} d\Omega' \int_0^\infty dE' v(E') \sum_r(\vec{r}, E', t) N_0(\vec{r}, E', \Omega', t) \quad (14)$$

$$\Lambda = \Lambda(t) = \frac{1}{V} \int_u du N_0^*(\vec{r}, E, \Omega) N_0(\vec{r}, E, \Omega, t) \quad (15)$$

$$\bar{P} = \bar{P} \quad (16)$$

$$\begin{aligned} \bar{P}_1 = \bar{P}_1(t) = \\ = \frac{\bar{P}_1}{V} \int_u du N_0^*(\vec{r}, E, \Omega) \frac{f_1(E)}{4\pi} \int_{\Omega'} d\Omega' \int_0^\infty dE' v(E') v' \sum_r(\vec{r}, E', t) N_0(\vec{r}, E', \Omega', t) \quad (17) \end{aligned}$$

$$F \Lambda c_i(t) = \frac{1}{4\pi} \int_{\Omega} du N_0^*(\bar{r}, E, \Omega) f_i(E) C_i(\bar{r}, t) \quad (18)$$

$$F \Lambda Q(t) = \int_{\Omega} du N_0^*(\bar{r}, E, \Omega) S(\bar{r}, E, \Omega, t) \quad (19)$$

Eqs. (11) and (12) are the conventional kinetics equations that were sought. In practice,  $P(t)$  is proportional to the reactor power,  $\rho$  is the reactivity,  $\Lambda$  the prompt neutron mean lifetime etc. It must be emphasized, however, that in the general case it is difficult to assign exact physical meanings to these quantities.

To see this clearly, notice that the normalization factor  $F$  is arbitrary. Consequently, even though the ratio  $\rho - \bar{\beta} / \Lambda$  is independent of  $F$ , the quantities  $\rho$ ,  $\bar{\beta}$  and  $\Lambda$  cannot be determined independently or be considered in general as physically meaningful quantities.

In fact, even if  $F$  were not arbitrary, still the exact calculation of  $\rho$ ,  $\bar{\beta}$  and  $\Lambda$  would require knowledge of the time dependent neutron density. Since this is difficult, approximations must be introduced and therefore the conventional equations are approximate bilinear averages.

For a more complete discussion of the limitations and the experimental interpretation of the conventional kinetics equations, the reader is referred to the literature (1-5).

In spite of the fact that the conventional form of the kinetics equations is approximate, it is extremely useful for a variety of qualitative dynamic analytical studies of reactor

systems. This topic is discussed in the next section.

#### 4. ANALYSIS OF REACTOR DYNAMIC BEHAVIOR

##### 4.1 General Remarks

The degree to which Eqs. (11-12) are good analytical approximations of the dynamic behavior of a reactor depends on the accuracy with which the quantities  $(\rho-\beta)/\Lambda$  and  $\beta_1/\Lambda$  can be computed. In practice, the quantities  $\beta/\Lambda$  and  $\beta_1/\Lambda$  are usually assumed as constant\* and assigned values, derived either from the inhour equation through period measurements or from transfer function measurements. It turns out that assuming  $\beta_1$  and  $\Lambda$  constant does not lead to intolerable discrepancies in a large variety of practical problems.

If  $\beta_1, \Lambda$  are assumed as constant, the factor that gives rise to time dependent variations of the reactor parameters (power, temperature, pressure etc.), from the standpoint of analysis, is the reactivity  $\rho$ .

The analytical understanding of the dynamic behavior of the reactor is therefore connected with the satisfactory answer of the following three questions:

- a. What are the physical processes or mechanisms that affect the reactivity and how can they be represented analytically?
- b. Given these mechanisms and their analytical expressions, what is the time dependence of the power level and other important reactor parameters?

\*Actually, for most of the mathematical procedures that will be presented later this requirement is not necessary and can be relaxed to that of bounded and uniformly convergent time dependent functions.

c. Given a reactivity variation, is the reactor power tolerably bounded or does the power increase indefinitely?

It is the purpose of this section to touch briefly on the first question with particular emphasis on the mechanisms that are important to reactor safety namely the mechanisms that are usually referred to as shutdown mechanisms. Questions (b) and (c) are discussed in another part of these notes.

#### 4.2 Reactivity Effects

Inspection of Eq. (13), that is the formal definition of reactivity, clearly indicates that reactivity can be influenced by the following factors.

- a. The neutron spectrum
- b. Variations of the fission, scattering and absorption macroscopic cross sections.
- c. The spatial neutron distribution.

These factors are in turn dependent on the materials of the reactor such as fuel, coolant, moderator and structure, on the temperature and temperature distribution in the reactor, the coolant flow and pressure, the voids and their distribution etc.

Reactivity variations can be stimulated either by changes in the material composition of the reactor or by changes in the macroscopic parameters that characterize the physical state of the reactor. These changes may be intentional (for control purposes) or accidental.

Equation (13) is difficult to use in its present form for analytical studies of reactor dynamics. In practice one usually approximates Eq. (13) by linear or nonlinear functions or functionals of the reactor power or of representative reactor parameters such as temperatures, pressures etc. For example

$$\rho(t) = \int_0^t g(t-\tau)P(\tau)d\tau \quad (20)$$

or

$$\rho(t) = \sum_{i=1}^k r_i T_i(t) + \sum_{j=1}^k r_j' T_j^2(t) + pP(t) + f(t) + \dots$$

where  $g(t)$  is a known kernel,  $T_i(t)$ 's are representative reactor parameters, solutions of equations of the form

$$\frac{d}{dt} T_i(t) = f_i(T_j(t), P(t)) \quad i, j = 1, 2, \dots, k \quad (21)$$

and  $f(t)$  represents any externally imposed variation of reactivity (for example control rod motion, poison concentration etc).

Equations (21) are generally derived from the partial differential equations that describe the physical processes that take place in the reactor such as heat transfer, fluid flow etc. The derivation is achieved by means of averaging techniques similar to the ones used for the conventional form of the neutron kinetics equations.

The coefficients  $\beta_1, \beta_2, \beta_3, \beta_4$  are usually referred to as temperature, pressure or power coefficients of reactivity and the effects are classified as temperature, pressure or power reactivity effects respectively.

The particular analytical approximation of the reactivity effects depends on the reactor type and the magnitude and rate of change of the reactor parameter variations under consideration. That the approximation should be dependent on the reactor type is self evident since different physical processes occur in different reactors. On the other hand the dependence on the magnitude and rate of change of the variations is an implicit dependence which arises from considerations of the degree of approximation rather than the magnitudes and rate of change as such. For example, consider a heterogeneous reactor with a Doppler coefficient of reactivity. If the rate of rise of the reactor power level is small, the heat is readily transferred to the coolant, the temperature of the fuel does not change appreciably and the Doppler reactivity is so small that it can be neglected in the presence of other reactivity effects introduced by the coolant or the moderator. On the other hand, if the rate of rise of the reactor power is so fast that the fission heat released does not have a chance to be conducted to the coolant, the fuel temperature rises appreciably and the Doppler reactivity effect is predominant.

The detailed discussion of the approximations used in different reactors is outside the scope of these notes. Suffice it only to emphasize that whatever approximations are used they should not be based on "curve fitting" but on a thorough understanding of the physics of the problem and good evaluation of the existing experimental results.

What follows is a discussion of those reactivity effects that are primarily responsible for the safe operation of water reactors.

#### 4.3 SHUTDOWN MECHANISMS IN WATER REACTORS

##### 4.3.1 General Remarks

From the standpoint of safety, reactivity effects, inherent to the reactor, which tend to shut the reactor down under all credible, severe and externally applied reactivity disturbances, are called shutdown effects. The mechanisms responsible for these effects are called shutdown mechanisms.

In discussing shutdown mechanisms, in a qualitative manner, it is expedient to think in terms of first order effects and to consider various coefficients of reactivity. For example, suppose that a shutdown effect is associated with the average temperature  $T_A$  of the coolant and that for every degree change of this temperature there is  $A$  ( $\$/^{\circ}\text{C}$ ) reactivity introduced in the reactor. In other words the reactivity effect is

$$\rho_T = AT_A \quad (\$) \quad (22)$$

A is the temperature coefficient of reactivity. If the coefficient A were the only coefficient of reactivity in the reactor, it should be negative because then if the average temperature rises, due to some externally applied positive reactivity, this rise would introduce a negative reactivity which would tend to bring the reactor back to criticality and therefore to keep the reactor power limited.

The problem is similar if one considers several reactivity effects simultaneously but the arguments are not as easily tractable.

Actually, the magnitude and sign of the coefficients of reactivity of various effects are not adequate by themselves to give a good measure of the importance of a particular reactivity contribution as a shutdown effect. Even more important is the relative time sequence of events in the reactor. This time sequence can be described in several ways, depending on the method of approach to the problem of analysis of the dynamic behavior of the reactor. However, its meaning can be appreciated through the following example.

Suppose that a heterogeneous reactor has only one negative temperature coefficient of reactivity, associated with the average coolant temperature. Assume that the reactor is excited by an externally imposed step change of reactivity, leading to a rise of the nuclear heat rate. If the fuel is a poor heat conductor, it will take a relatively long time for the energy released in the fuel to get to the



coolant. Thus, the fuel temperature may rise to a destructive level before the coolant has a chance to introduce a negative reactivity effect. This shows that the negative coefficient becomes useless by the delayedness of its action.

In general the situation may be more involved than described above. The example, however, illustrates the importance of the time element on the effectiveness of various reactivity mechanisms.

In the context of the preceding remarks, the ideal shutdown mechanism is one that is characterized by a negative coefficient and acts promptly. The magnitude of the coefficient need not be large if it is promptly effective. The reason for this is that the reactivity investment, under normal operating conditions, is small while the shutdown properties are effective when mostly needed, namely at the beginning of a large disturbance.

The most effective shutdown mechanisms result from a heating of the fuel or the moderator by absorption of fission fragments, neutron energy and beta or gamma-rays. The reactivity effects resulting from these processes are usually much smaller than others such as voids due to boiling. However, they are so prompt that they are more important than the other large effects with long time delays.

Unfortunately, it is still difficult to analytically assess the exact value and promptness of various shutdown

effects. This is partly due to the fact that in a reactor, several reactivity inducing processes occur simultaneously and it is not always easy to devise experiments which allow the isolation of the various mechanisms. In addition, most of the shutdown effects are nonlinear. In spite of this difficulty several reliable mechanisms have already been confirmed, and are subsequently discussed.

#### 4.3.2 The Doppler Effect

A very important shutdown mechanism is due to the broadening of absorption resonances due to the fuel temperature rise. This is called the Doppler effect.

The Doppler coefficient of reactivity is very small and its sign is often difficult to establish theoretically. However, for low enrichment,  $UO_2$ -fuel cores it has been experimentally shown that the Doppler effect is an effective shutdown mechanism. (6-8)

Experimentally it is found that the Doppler coefficient of reactivity is a nonlinear function of the energy content of the fuel

$$\delta \propto - \frac{1}{E^{0.26}} \quad (23)$$

where E is the energy. Values in the range of 2-3%/MW-sec have been measured. (7)

Theoretically, the same coefficient is estimated to be inversely proportional to the square root of the energy (9)

$$\delta \propto - \frac{1}{\sqrt{\alpha E_0 + E}} \quad (24)$$

where  $\alpha$  is a constant depending on the heat capacity and temperature of the fuel,  $E_0$  the initial energy content of the fuel.

In the case of very fast transients and if only the Doppler effect is effective, the step response of the reactor is adequately represented by

$$\frac{d\beta}{dt} = (\alpha_0 - bE^n) \frac{dE}{dt} \quad (25)$$

$$\beta = \frac{dE}{dt} \quad \alpha_0 = \text{initial period}$$

$n$  = constant (experimentally found equal to 0.74).

This equation can be readily integrated for  $n=b/c$ ,  $b, c$  integers and gives good correlations with experimental data.

(10)

#### 4.3.3 Spectral Shift Effect

Another prompt shutdown mechanism is due to the change of the neutron spectrum caused by heating of the hydride fuel elements of hydride-fueled, water moderated reactors. This may be called the spectral shift effect and it has been experimentally and theoretically investigated in the TRIGA reactors. (11)

The prompt spectral shift effect is due to a change in the neutron spectrum caused by the heating of the ZrH of the UZrH fuel elements. When the fuel is heated, the neutrons gain energy from the excited states of hydrogen and thus escape faster from the fuel. The end result of this is that the capture in the water moderator is increased and a negative reactivity is introduced.

The coefficient of the spectral shift reactivity effect is found both experimentally and theoretically to be of the order of  $(-1.5\%/^{\circ}\text{C})$ . This coefficient can be used in connection with Fuchs model (12) to predict the burst behavior of the reactor.

#### 4.3.4 Metal-Plate Expansion Effects

Thermal expansion of the fuel plates of heterogeneous water reactors is also a good shutdown mechanism because it acts very promptly during severe accidents.

This mechanism is useful in an overmoderated reactor since the expansion of the fuel results in the expulsion of the moderator and therefore a reduction of the reactivity.

The SPERT III tests have demonstrated the importance of fuel plate expansion, even though the results were not conclusive. (13)

#### 4.3.5 Other Effects

There are many other mechanisms which may contribute to the inherent shutdown capabilities of a water reactor.

Radiolytic gas may be formed by the irradiation of the water by the neutron or other reactor radiations. The reactivity effects of this mechanism have been studied in the KEWB reactor. (14) The energy coefficient of reactivity for this reactor ranges from 0.5-2.5  $\$/\text{MW-sec}$ . This rather large coefficient, however, is not adequate by itself to guarantee safety. The reason is that the volume of the promptly formed radiolytic gas is relatively small and does

not become effective reactivity-wise until it expands through bulk heating. The latter process involves time delays which, as already discussed, reduce the shutdown effectiveness of the large coefficient. Of course, if the water is heated by direct neutron, gamma- or beta ray interactions, the effectiveness of the radiolytic gas may be increased faster. In this case, the direct heating of the water itself contributes to the shutdown capabilities of the reactor.

Another mechanism, similar to the radiolytic gas formation is the formation of steam. But steam formation also involves long delays and consequently is not reliable for safety against large reactivity insertions.

Lately, several investigators are seriously considering gas microbubble dilation as a possible mechanism contributing to reactor shutdown. (15-16) The sources of the microbubbles are minute cavities in the metal fuel surfaces and radiolytic gas formed in the bulk of the water but migrating to the surface by Brownian motion. During a power excursion the microbubbles on the surface expand in response to: a) the surface temperature rise, b) evaporation of water into the gas and, c) decrease of surface tension with increasing water temperature.

This mechanism may account for some discrepancies in the SPERT III data. (16)

NOMENCLATURE

$C_i(\vec{r}, t), c_i(t)$	delayed neutron precursor concentrations
$E$	neutron energy
$f_i(E) \quad (i=0,1,\dots,m)$	prompt and delayed neutron energy spectrum
$N(\vec{r}, E, \Omega, t)$	neutron density in the $(\vec{r}, E, \Omega, t)$ - space
$S(\vec{r}, E, \Omega, t)$	source density in the $(\vec{r}, E, \Omega, t)$ - space
$v$	neutron speed
$v\Omega$	neutron velocity
$\beta_{1,2}$	delayed neutron fractions
$\lambda_1$	delayed neutron decay constants
$\nu(E)$	number of neutrons per fission, of energy $E$
$\Sigma_f(\vec{r}, E, t)$	fission cross section
$\Sigma_s(\vec{r}, E, t)$	scattering cross section
$\Sigma_t(\vec{r}, E, t)$	total cross section

REFERENCES

1. A.F. Henry, The Application of Reactor Kinetics to the Analysis of Experiments, Nuclear Sci. and Eng'g., 3, 52-70, (1958).
2. E.R. Cohen, Some Topics in Reactor Kinetics, Proc. 2nd Intern. Conf. Peaceful Uses of Atomic Energy, 11, 303-309, (1958).
3. J. Lewins, The Basis and Application of Perturbation Theory to Neutron Diffusion, ScD Thesis, MIT, June (1959)
4. S. Kaplan, The Property of Finality and the Analysis of Problems in Reactor Space-time Kinetics by Various Modal Expansions, Nuclear Sci. and Eng'g., 9, 357-361, (1961).
5. E.P. Gyftopoulos and J. Devvoght, On the Range of Validity of Nonlinear Reactor Dynamics, Nuclear Sci. and Eng'g., 10, 372-376, (1961).
6. A.H. Spano et al, Self Limiting Power Excursion Tests of a Water Moderated, low enrichment  $UO_2$  Core in Spert I, IDO 16751, Feb. (1962).
7. A.H. Spano and W.K. Ergen, Direct Measurement of the Dynamic Coefficient by Self-limiting Power Excursion Tests, Trans. Amer. Nucl. Soc., 5, 1, (1962) p. 157.
8. P. Greebler and D.B. Shever, Excursion Shutdown Mechanism for Large Fast Oxide Reactors, Trans. Amer. Nucl. Soc., 5, 1, (1962) p. 158.
9. A.H. Spano (Private Communication).
10. E.P. Gyftopoulos, Class Notes for 22.242 at MIT.
11. G.R. Hopkins et al, The TRIGA Shutdown Mechanism, Trans. Amer. Nucl. Soc., 5, 1, (1962) p. 159.
12. K. Fuchs, Efficiency for Very Slow Assembly, LA596, Aug. 2, (1946).
13. R.W. Garner, Non-boiling Reactivity Compensation in SPERT III Power Excursions, Trans. Amer. Nucl. Soc. 4, 1, (1961) p. 40.
14. C. Bumpus et al, Measurement and Interpretation of Dynamic Void Growth in Homogeneous Fissile Solutions, Trans. Amer. Nucl. Soc., 4, 1 (1961) p. 71.

15. H. Kouts (BNL) (Private Communication).
16. S.M. Zivi, Gas Microbubble Dilation - A Reactor Shutdown Mechanism, Trans. Amer. Nucl. Soc., 5, 1 (1962) p. 161.

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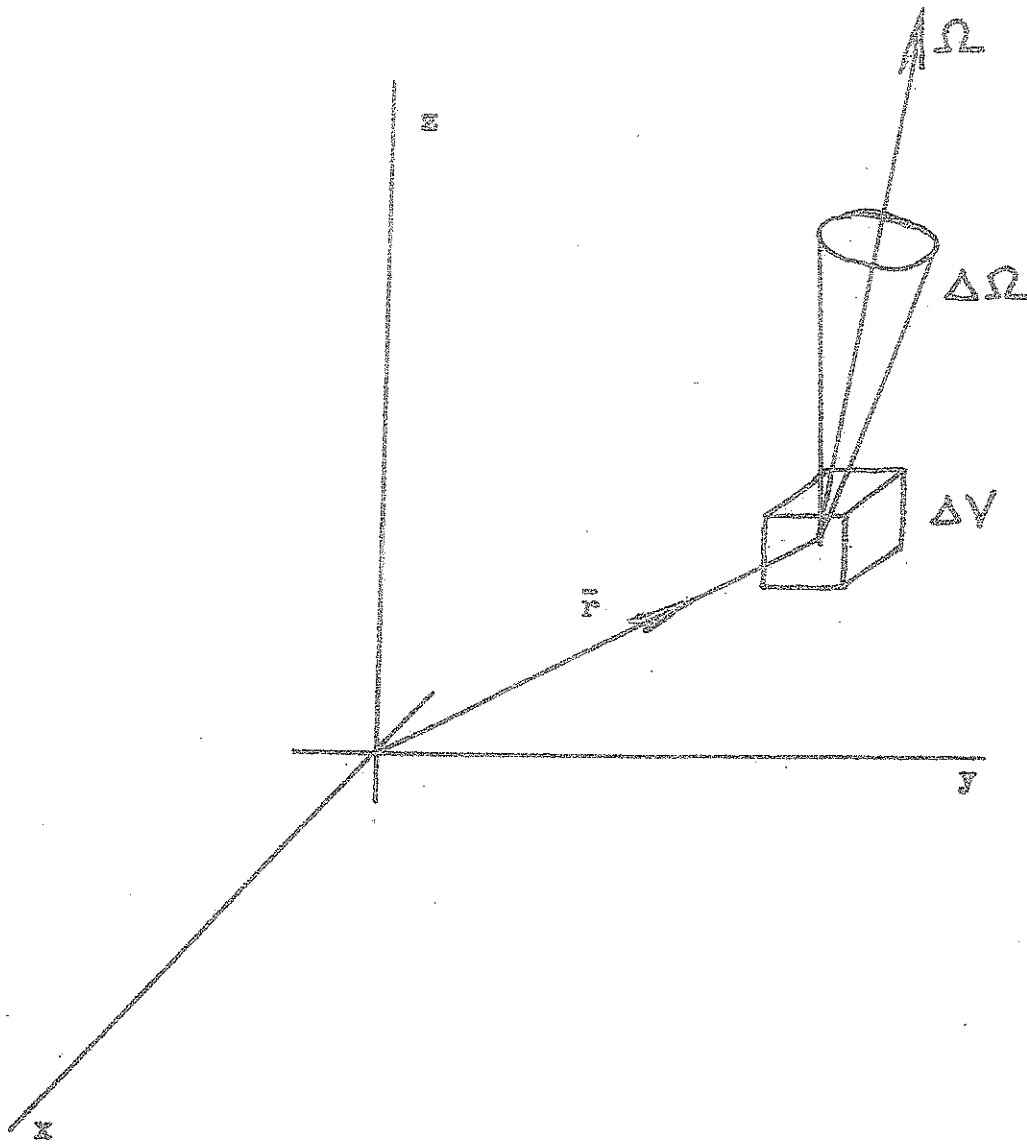


Figure 1