2-5-55.

Elias

This is an excellent job.
Hope you enjoyed it.

John Trump

ON THERMIONIC EMISSION

by

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INDEX

ABSTRACT

The present paper is a quick survey of the phenomenon of thermionic emission.

The first part of it deals with the experimental data pertinent to thermionic emission and the early attempts of different investigators for a sound scientific explanation of them.

In the second part an attempt is made to present the Fermi Statistics which is the basis of the Fowler-Nordheim theory of electron escape through a potential barrier. The emphasis of the paper is exactly on this statistics because even though all explanations of thermionic emission agree with the experimental evidence, only the Fowler-Nordheim theory lies within the boundaries of modern conceptions of matter as they have been developed by Pauli, Sommerfeld and Bloch.

In the third part the details of the derivation of the thermionic emission formula are presented, based on the Fermi Statistics.

The paper is closed by a brief discussion of the general tendency of Modern Science to unify the explanation of the physical world, with the least number of principles and assumptions.

PART I

1.1 EARLY HISTORY

Two hundred years ago many scientists noticed that air loses its insulating strength when in contact with the surface of a hot metal. Some work devoted to the study of the phenomenon proved that this was due to emission of charged particles from the surface of the hot metal. This is what one calls thermionic emission.

Particularly, in 1883, Edison noticed a transport of electricity across a relatively good vacuum within the bulb of a carbon filament electric lamp when a positive plate was introduced in the bulb.

Later Fleming, using Edison's observations, constructed a rectifier valve.

The precise nature of the carriers of electricity, emitted from hot metals in vacuum, was revealed by Thomson in 1889.

1.2 FUNDAMENTAL THERMIONIC PHENOMENA

When the phenomenon of thermionic emission was established, different investigators tried to study its dependence on the temperature of the emitter, the voltage of the collecting electrode and the kind of material used as an emitter.

Using the basic circuit shown in Fig. 1, they established the results shown schematically in Fig. 2 and 3.

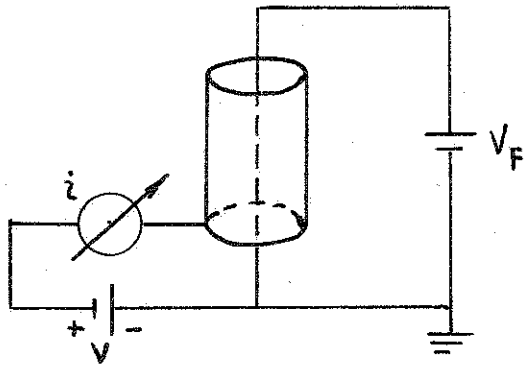

FIG. 1

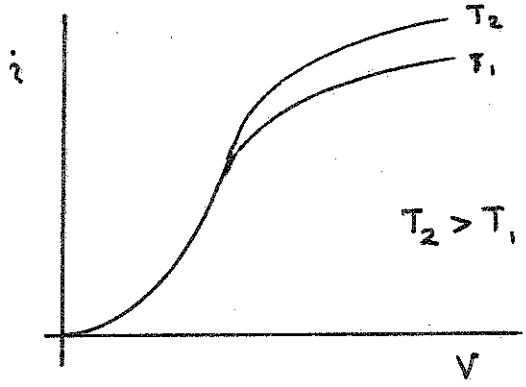

FIG. 2

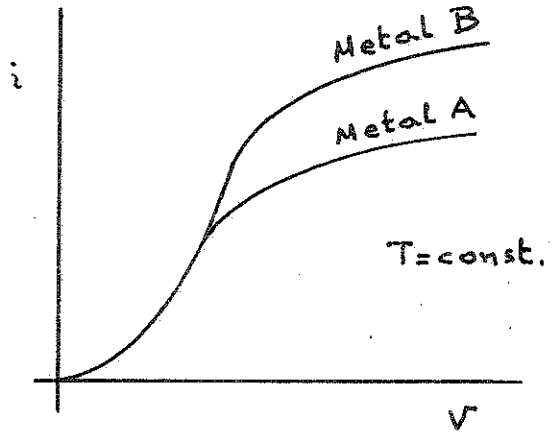

FIG. 3

The next natural step was the theoretical justification of the results and their quantitative representation.

The function $i = f(V,T)$, where T is considered as a parameter, presents two distinct parts:

a) The sub-saturation part, which has been adequately explained by Child[1], as due to space charge effects and is of no interest of this paper.

b) The saturation part. Many attempts have been made to justify theoretically the dependance of the saturation current on temperature. The experimental results were satisfactorilly approximated by the function

$$I_s = AT^n e^{-\frac{x}{\kappa T}} \qquad (1.1)$$

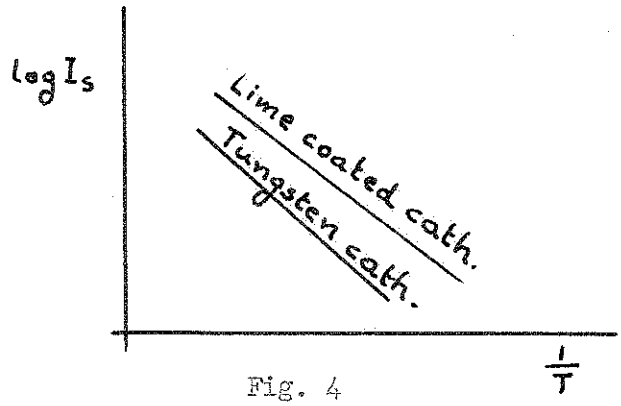

Fig. 4

where A = constant

$n = \frac{1}{2}$ or 2

x = constant depending on the material. Even though this may seem impossible ($n = \frac{1}{2}$ or 2) at a first glance, it is true on account of the rapid change of the function $e^{-\frac{x}{\kappa T}}$ with respect to T^n when n is of the order of 1.

From an engineering point of view both answers are acceptable. The scientifically correct formula depends though on the actual phenomena by which thermionic emission takes place.

First Richardson[5] attempted an evaluation of the saturation current based on Drude's classical assumption that the electrons of a

metal behave like a perfect gas. Richardson assumed further that these electrons have a Maxwellian distribution of velocities and that at the surface of the metal there's a potential barrier of C volts. Thus he determined a saturation current

$$I_s = AT^{1/2} e^{-C/\kappa T} \qquad (1.2)$$

After Richardson, Wilson [9] showed that an emission formula could also be obtained, based on a thermodynamic method involving Clapeyron's equation. This idea was developed by both Wilson and Richardson[6]. They assumed that the electronic gas which surrounds the emitter is in thermal equilibrium, even though electronic current is flowing through it.

Thus they found that the saturation current is

$$I_s = \bar{D} \; AT^2 \, e^{-\frac{x}{\kappa T}} \qquad (1.3)$$

where

$\bar{D}$ = the mean transmission coefficient

x = the work function

In the above formulas the constants are evaluated from plots like the one shown in Fig. 4.

This was the situation until a few decades ago when it became apparent that Drude's conception of solids was not true even though the idea of an electronic gas is still very appealing.

The objection to Drude's ideas is that they lead to some conclusions which are not experimentally verified. (See for example Seitz: Theory of Solids p. 139 - 1940)

On the other hand Sommerfeld, Pauli and Bloch had given a picture of the structure of metals based on Fermi-Dirac Statistics which gave

wonderful results in explaining thermal and electrical conductivity and other experimental data referring to the behavior of metals.

So if any attempt, of relating the thermionic emission to the metal structure, was to be made, it ought to be based on the modern conceptions of matter. The introduction of new assumptions seemed both superfluous and undesired.

This has been tried very successfully by Nordheim[4] and Fowler[3] who by using Fermi Statistics found that the saturation current is given by the formula

$$I_s = DAT^2 e^{-\frac{\epsilon \phi}{\kappa T}} \qquad (1.4)$$

ϕ = work Function in eV

The Fowler Nordheim expression for the saturation current is similar to Richardson's equation (1.3) but it is based on more sound assumptions which are in perfect agreement with the modern conceptions of solids.

Here instead of dealing further with the details of the derivation of the Fowler-Nordheim equation, we pause for a while and present the Fermi Statistics.

The reason why we present the Fermi Statistics, apart from its scientific value, is that in almost all the literature it is referred as "according to Fermi Statistics one has.....", so it seemed rather interesting to investigate what exactly that Fermi Statistics is.

PART II

2.1 FERMI STATISTICS

When Fermi[2] proposed his theory of quantization of the molecules of ideal gases, his main objective was the explanation of the behavior of ideal gases at very low temperatures. This theory, though, had a tremendous impact on the further development of the modern conceptions of matter.

Fermi started by saying that he would try to justify the behavior of ideal gases by using the least number of assumptions.

Thus he postulated that the molecule of an ideal gas may be assumed as enclosed in a small parallelepiped which moves back and forth in a periodic manner so that it can be quantized.

The hypothesis that the gas is ideal allows one to neglect the effect of the other molecules on the one under consideration, so that the motion of the molecule is determined only by external forces.

It is worth noting here that this is generally a weak point of the theory.

Next, Fermi examined whether Pauli's principle of exclusion could be applied in the case of ideal gases.

He assumed that only one molecule in a given gas can have a given set of quantum numbers. As quantum numbers he considered not only the ones which characterize the internal motions of the molecule but also the ones which refer to its translational motion.

First he assumed that the molecule is under the influence of an external force, such that it is performing a periodic motion, which does not depend on the value of the force. This of course can be done in

different ways, one of which is to consider the molecule as a harmonic oscillator under the influence of a central force. If v is the characteristic frequency of the oscillator this force will be

$$4\pi^2\nu^2 m r$$

and its potential energy

$$u = +2\pi^2\nu^2 m r^2 \qquad (2.1)$$

r distance of molecule from the center of the force.

The quantum numbers of the molecular oscillator will be $s_1, s_2 \cdots$ In order to completely characterize a particular gas molecule one should also consider the quantum numbers which refer to its internal motions. But one can avoid this complication, by examining only monatomic gases which are in a ground state and so there is no need for internal quantum numbers. Fermi stated Pauli's principle in this case as follows:

"In a given amount of monatomic ideal gas there can exist utmost one molecule with a prescribed set of quantum numbers $s_1, s_2 \cdots$"

The total energy of this molecule will obviously be

$$w = h\nu (s_1 + s_2 + \cdots) = hs\nu \qquad (2.2)$$

or in other words the total energy can be only an integer multiple of $h\nu$. Of course this integer can be realized by different combinations of $s_1, s_2 \cdots$ as long as $s_1 + s_2 + \cdots = s$, where s_i can take any value 0, 1, 2---. The number of possible combinations is

$$Q_s = \frac{(S+1)(S+2)}{2} \qquad (2.3)$$

in other words there are :

1 molecule with energy 0

3 molecules with energy $h\nu$

6 molecules with energy $2h\nu$

$\frac{(S+1)(S+2)}{2}$ " " " $sh\nu$

It is interesting to note here that at zero temperature ($T = 0$) a given amount of gas will be at its lowest level of energy.

If it has only one molecule this would be at zero energy level.

If it has 4 molecules, one will be at the zero level and 3 will occupy the $h\nu$ energy level.

Last if it has 10 molecules, they will be distributed as follows:

1 at zero energy level

3 at $h\nu$ " "

6 at $2h\nu$ " "

Fermi noticed this distribution and in his paper he emphasizes the similarity between an ideal gas and the electrons of an atom with many electrons (at $T = 0$). This hint may have been the ignition spark of the work of Sommerfeld and others on solids and of the work of Fowler-Nordheim on thermionic emission.

Gases are not though made of 1, 4, 10... molecules. So Fermi examined how a given amount of energy is distributed between n_o molecules of an ideal gas which occupy a unit volume. The total amount of energy is:

$$W = E h\nu \qquad (2.4)$$

Suppose that N_s is the number of molecules having energies $sh\nu$

According to the previous assumptions

$$\begin{aligned} N_s &\leq Q_s \\ \Sigma N_s &= n_o \\ \Sigma s N_s &= E \end{aligned} \qquad (2.5)$$

Let P be the number of combinations which is made of N_o places with energy 0, N_1 places with energy $h\nu$ ..., N_s places with energy $sh\nu$ Two combinations will be the same when the places occupied by the mole-

cules are the same. Thus two combin ations which differ only by a permutation of the place of their molecules they must be considered as identical. In case such combinations would be considered as different, then the number P should be multiplied by $n_o!$ From what follows though one can see very easily that this would have no effect on the final result, what-so-ever.

According to what has been said above, the number of combinations of the N_s molecules, out of the Q_s possible positions for the energy $sh\nu$, will be given by

$$\binom{Q_s}{N_s} = \frac{Q_s\,!}{N_s!\ (Q_s - N_s)\,!} \tag{2.6}$$

In other words this is the probability of having N_s molecules at the energy level $sh\nu$. Thus the total probability of having N_o molecules at 0 level, N_1 at the $h\nu$ level e.t.c., will be

$$P = \binom{Q_o}{N_o} \times \binom{Q_1}{N_1} \times \cdots \binom{Q_s}{N_s} = \prod \binom{Q_s}{N_s}$$

since all energy levels occur simultaneously.

The next problem is to find the most probable value of P under the constraints

$$\begin{aligned} \Sigma N_s &= n_o \\ \Sigma s N_s &= E. \end{aligned} \tag{2.7}$$

This is a maximum problem solved with the help of Lagrange's coeffi-cients and Stirling's formula for the factorials.

Fermi showed that P has its most probable value when

$$N_s = Q_s \frac{a e^{-\beta s}}{1 + a e^{-\beta s}} \tag{2.8}$$

where a, β constants to be determined from n_o and E

since

$$n_o = \sum_o^\infty Q_s \frac{a e^{-\beta s}}{1 + a e^{-\beta s}}$$

$$\frac{W}{h\nu} = E = \sum_o^\infty s Q_s \frac{a e^{-\beta s}}{1 + a e^{-\beta s}} \qquad (2.9)$$

The theory which has been presented in the previous pages would be acceptable if it were in agreement with the experimental evidence. A step towards this end is the evaluation of the function $T = f(a, \beta)$ which correlates the temperature of the ideal gas with its energy and volume.

This could be done of course by assuming Boltzmann's entropy principle as true. According to this principle

$$S = \kappa \log P$$

$$T = \frac{dW}{dS} \qquad (2.10)$$

But such an assumption would mean one extra statement in the theory, a fact which is altogether undesirable.

Fermi noticed instead, that the temperature can be related to the energy of a given amount of ideal gas through the density, which is a function of the distances between the molecules.

Thus when $r \to \infty$ the previously developed formulae must coincide with the classical ones since the motions are completely randomized. For $r \to \infty$ the average kinetic energy of the molecule is $\frac{3}{2} \kappa T$ and the distribution of velocities is Maxwellian. That means that the temperature function is known for densities $\to 0$.

This point is very essential in the Fermi Statistics because it

is the only reason why classical statistics is identical with Fermi Statistics, at very high temperatures. To the author's astonishment, this fact is presented as a happy coincidence in many a reference.

For higher densities Fermi examined the probability of molecules with Kinetic energies between L and $L + dL$. The total energy of such molecules is

$$L + 2\pi^2\nu^2 m v^2 \qquad L + 2\pi^2\nu^2 m v^2 + dL$$

and since

$$sh\nu = L + 2\pi^2\nu^2 m v^2$$
$$ds = \frac{dL}{h\nu} \tag{2.11}$$

A molecule characterized by the quantum numbers $s_1, s_2 \cdots$ will have the coordinates

$$x = \sqrt{Hs_1}\cos(2\pi\nu t - a_1)$$
$$y = \sqrt{Hs_2}\cos(2\pi\nu t - a_2) \tag{2.12}$$
$$z = \sqrt{Hs_3}\cos(2\pi\nu t - a_3)$$

where
$$H = \frac{h}{2\pi^2\nu m} \tag{2.13}$$

a_i = phase constants with equal probability of occurrence.

But the probability of having x, y, z between $x + dx \div x$ $y + dy \div y$ $z + dz \div z$ is

$$\frac{dx\,dy\,dz}{\pi^3\sqrt{(Hs_1 - x^2)(Hs_2 - y^2)(Hs_3 - z^2)}} = \frac{dx\,dy\,dz}{\pi^3\sqrt{\Gamma}} \tag{2.14}$$

and the total probability for all possible combinations is

$$\frac{1}{Q_s}\,\frac{dx\,dy\,dz}{\pi^3} \sum \frac{1}{\sqrt{\Gamma}}$$

where the sum is to be taken for all integer solutions of

$$S = S_1 + S_2 + S_3$$

If one assumes that there are N_s molecules with energy $s h\nu$, which follow the Fermi statistics, this probability becomes

$$dn_s = \frac{a e^{-\beta s}}{1 + a e^{-\beta s}} \frac{dx\,dy\,dz}{n^3} \Sigma \frac{1}{\sqrt{r}} \tag{2.15}$$

Integrating over the whole spectrum of positions one finds

$$n_s = \frac{2}{n^2 H^2} \frac{a e^{-\beta s}}{1 + a e^{-\beta s}} \sqrt{Hs - r^2} \tag{2.16}$$

Thus $n_s ds$ of these molecules may be at the energy level between L and L $+dL$. Hence the density of molecules at this level of energies will be

$$n(L)\,dL = n_s\,ds = \frac{2n(2m)^{3/2}}{h^3} \sqrt{L}\,dL \frac{a e^{-\frac{2n^2 \nu m \beta r^2}{h}} e^{-\frac{\beta L}{h\nu}}}{1 + a e^{-\frac{2n^2 \nu m \beta r^2}{h}} e^{-\frac{\beta L}{h\nu}}} \tag{2.17}$$

This formula coincides with the one given by Maxwell when $r \to \infty$

Thus

$$n^x(L)\,dL = k \sqrt{L}\,dL\, e^{-\frac{L}{kT}} = n(L)\,dL \quad (r \to \infty) \tag{2.18}$$

which means that

$$\beta = \frac{h\nu}{kT} \tag{2.19}$$

and the distribution expression now becomes:

$$n(L)\,dL = \frac{2n(2m)^{3/2}}{h^3} \sqrt{L}\,dL \frac{A e^{-\frac{L}{kT}}}{1 + A e^{-\frac{L}{kT}}} \tag{2.20}$$

$$A = a e^{-\frac{2n^2 \nu^2 m r^2}{kT}}$$

The total number of molecules is

$$n = \int_0^\infty n(L)\,dL = \frac{(2nmkT)^{3/2}}{h^3} F(A) \tag{2.21}$$

$$F(A) = \frac{2}{\sqrt{n}} \int_0^\infty \frac{A \sqrt{x}\, e^{-x} dx}{1 + A e^{-x}} \tag{2.22}$$

The average kinetic energy of the molecule is

$$\bar{L} = \frac{1}{n}\int_0^\infty L\, n(L)\, dL = \frac{3}{2} kT \frac{G(A)}{F(A)} \qquad (2.23)$$

$$G(A) = \frac{3}{4\sqrt{\pi}} \int_0^\infty \frac{A x^{3/2} e^{-x} dx}{1 + A e^{-x}} \qquad (2.24)$$

Thus the temperature is related to the physical condition of the gas because $A = f_1(T)$ and $\bar{L} = f_2(T)$ and consequently

$$p = \frac{2}{3} n \bar{L} = nkT \frac{G(A)}{F(A)} \qquad (2.25)$$

Next Fermi evaluated the integrals G (A) and F (A) and the specific heat of an ideal gas $c_v = \frac{d\bar{L}}{dT}$, which he finds in perfect agreement with the experimental evidence.

This is though out of the scope of this paper, once the main points of the theory have been emphasized and the assumptions which led to the development of Fermi Statistics presented.

Now is the appropriate time to examine how Fowler and Nordheim used the Fermi Statistics in their theory of thermionic emission.

PART III

3.1 THE FERMI ELECTRON GAS

Following Drude, assume that the electrons of a metal behave like an ideal gas. Further assume that the electronic potential energy inside the metal is constant (this implies no loss of generality whatso-ever) and equal to $-W_a$ and that it varies in a certain fashion up to zero, at infinite distance from the metal surface. The way in which the potential barrier varies from $-W_a$ to 0 need not concern us at the present moment.

The total energy of a given electron will be

$$E = L - W_a \qquad L = E + W_a \qquad (3.1)$$

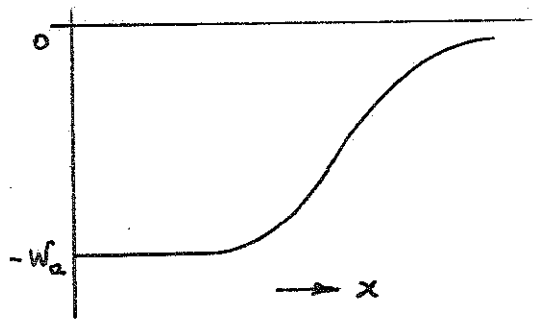

Fig. 5

Assuming further that these electrons follow Fermi Statistics their energy distribution curve will be given by the function (2.20) namely

$$dn = n(L)\,dL = C'\sqrt{L}\,dL\,\frac{1}{e^{\frac{L-L'}{kT}}+1}$$

where

$$C' = 2\,\frac{2\pi(2m)^{3/2}}{h^3} \qquad (3.2)$$

since electrons have an extra quantum number referring to their spin. and

$$e^{-\frac{L'}{kT}} = \frac{1}{a}\,e^{\frac{2\pi^2\nu^2 m r^2}{kT}} \qquad (3.3)$$

It is interesting to examine the meaning and value of the parameter L' for different particular cases.

a) Temperature equal to zero.

The form of $\frac{1}{e^{\frac{L-L'}{kT}}+1}$ is illustrated in Fig. 6. Thus one may write

$$dn = \begin{cases} C'\sqrt{L}\,dL & 0 \leq L \leq L'_o \\ 0 & L'_o < L \end{cases} \qquad (3.4)$$

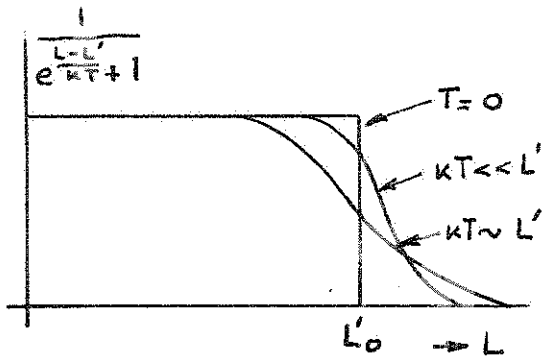

Fig. 6.

In other words at absolute zero all the energy levels below L'_o are filled with electrons whereas above L'_o they are completely empty. The actual value of L'_o may be determined from the relationship

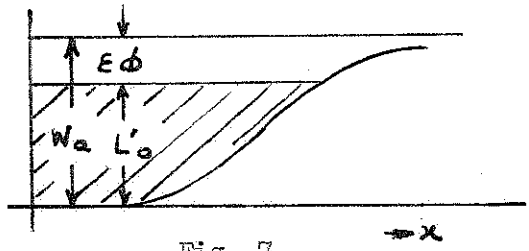

Fig. 7

$$\int dn = n_o = C' \int_0^{L'_o} \sqrt{L}\,dL = C' \frac{2}{3} L'^{3/2}_o \qquad (3.5)$$

where n_o is the number of electrons per unit volume of electronic gas. Thus we find

$$L'_o = \frac{h^2}{2m} \left(\frac{3n_o}{8\pi}\right)^{2/3} \qquad (3.6)$$

It is interessting to note that L'_o depends only on the density of the particles.

In the previous derivations we tacitly assumed a continuous distribution of particles and energies which is not a loss of generality, if one considers the smallness of distance between the particles and the small steps by which energy levels change their values.

The difference between W_a and L'_o is what we call the work function of the metal.

Also the mean energy of the particles, a quantity useful for comparisons, is in this case

$$\bar{L}'_o = \frac{1}{n_o} \int_o^{L'_o} C' \sqrt{L}\, dL = \frac{3}{5} L'_o \qquad (3.7)$$

b) Case in which T is small in comparison with L'_o.

This case is very interesting because L'_o is much larger than κT below the melting point of most metals.

The parameter L' may be determined from the equation

$$n_o = C' \int_o^{\infty} \frac{\sqrt{L}\, dL}{e^{\frac{L-L'}{\kappa T}} + 1} \qquad (3.8)$$

Sommerfeld[8] and Bethe developed a convenient method for evaluating integrals of this type. The derivation is not of any interest to us. The result though is very meaningful. Namely

$$\begin{aligned} L' &\cong L'_o \left\{ 1 - \frac{\pi^2}{12} \left(\frac{kT}{L'_o} \right)^2 \right\} \\ \bar{L} &\cong \bar{L}_o \left\{ 1 + \frac{5}{12} \pi^2 \left\{ \frac{kT}{L'_o} \right\}^2 \right\} \end{aligned} \qquad (3.9)$$

where L'_o and $\bar{L}_o$ are given by (3.6) (3.7)

Needless to say that in case (b) L' can be approximated by L'_o.

c) Case in which L' is large and negative.

For most metals this is true at temperatures above the melting point. So this case is irrelevant to the purpose of this paper.

d) Intermediate case

This case also, in which κT is comparable to L', is not of any help to the discussion which will follow.

The derivations of cases a and b imply some very interesting conclusions. Namely that the assumption of an electronic gas, which follows Fermi Statistics, leads to plausible results which incidentally are in perfect agreement with the experimental evidence (to the extent the simpligying assumptions are true). On the other hand the parameter L' may be taken as equal to L'_o for all temperatures of interest in the case of thermionic emission, while from Fig. 6 we conclude immediately that for $T > 0$ there are electrons which have energies higher than L'_o.

3.2 THERMIONIC EMISSION AND SHOTTKY EFFECT ACCORDING TO FOWLER AND NORDHEIM

Fowler and Nordheim suggested that the emission of electrons from hot metals may be considered as the escape of energetic particles through a potential surface barrier.

They developed an interesting theory based on Fermi Statistics and Wave Mechanics (in order to take care of reflections of the escaping electrons) which agrees with the experimental data on thermionic emission.

In what follows we present a simplified version of this theory, neglecting altogether the affect of reflection.

It has been mentioned already that thermionic emission is the phenomenon in which an electronic current evaporates from a heated metal in the absence of an electric field, whereas Schottky[7] emission refers to the evaporation that occurs when the metal is at a negative potential.

Let us determine the way in which the electronic potential varies near the surface of a metal. Suppose that an electron is at a distance

x from the surface of an uncharged metal, where x is large compared with an interatomic distance and small compared with the dimensions of the surface. Then the only force that acts upon the electron is the classical attractive image force F_i which is given by the equation

$$F_i = - \frac{\varepsilon^2}{16\pi\varepsilon_0 x^2} \tag{3.10}$$

The potential energy associated with this force is clearly

$$V_i = - \frac{\varepsilon^2}{16\pi\varepsilon_0 x} \tag{3.11}$$

Assume further that inside the metal the potential is $-W_a$.

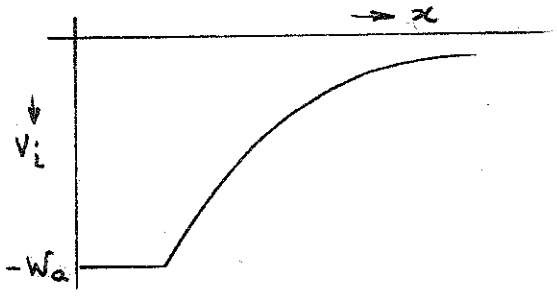

Fig. 8

This is not a restriction of the generality of the treatment which follows.

Of course there are some difficulties in the determination of the metal surface from the electric point of view, but they are of minor importance.

When the metal surface is at a negative potential, resulting in a repulsive field E, the potential energy associated with one charged particle is

$$V_e = - \varepsilon E x \tag{3.12}$$

$$L \geq W_a - \varepsilon\left\{\frac{\varepsilon E}{4\pi\varepsilon_0}\right\}^{1/2} + \frac{p_y^2 + p_z^2}{2m} = \tag{3.16}$$

$$= L_o' + \phi_E + \frac{p_y^2 + p_z^2}{2m} = L''$$

according to Fig. 7 and equation 3.15.

The total number of electrons with momenta in the range p_x $p_x + dp_x$.. e.t.c. striking a unit area of the surface, in unit time, is

$$\nu = \int n'(L)\, dp_x\, dp_y\, dp_z\, v_x = \int_v n'(L) \frac{\partial L}{\partial p_x} dp_i \tag{3.17}$$

since

$$v_x = \frac{\partial L}{\partial p_x}$$

where n'(L) is the number of electrons per unit volume of momentum space. The electronic gas under consideration follows, presumably, Fermi Statistics. Hence

$$n(L)\, dL = C' \sqrt{L}\, dL \frac{1}{e^{\frac{L-L'_o}{kT}} + 1} \tag{3.18}$$

The number of electrons given by the above expression occupy a spherical shell of radius p = const and thickness dp. The volume of this shell is

$$4\pi p^2 dp = 4\pi (2mL)\, d\sqrt{2mL} = 2\pi (2m)^{3/2} L\, dL \tag{3.19}$$

and thus the number of particles per unit volume of momentum space will be

$$n'(L) = \frac{2}{h^3} \frac{1}{e^{\frac{L-L'_o}{kT}} + 1} \tag{3.20}$$

and

$$\int n'(L) v_x\, dp_x\, dp_y\, dp_z = \frac{2}{h^3} \int \frac{1}{e^{\frac{L-L'_o}{kT}} + 1} \frac{\partial L}{\partial p_x} dp_x dp_y dp_z \tag{3.21}$$

and the total potential barrier at the surface of the metal takes the form of Fig. 9, described by the function

$$V_t = \begin{cases} -\dfrac{\varepsilon^2}{16\pi\varepsilon_0 x + \varepsilon^2/W_a} - \varepsilon E x & x \geq 0 \\ -W_a & x < 0 \end{cases} \qquad (3.13)$$

with a maximum value

$$V_m = -\sqrt{E}\,\frac{\varepsilon^{3/2}}{\sqrt{4\pi\varepsilon_0}} \qquad (3.14)$$

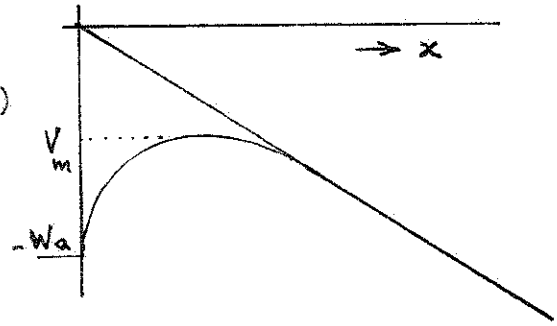

Fig. 9

This maximum value determines a new effective work function ϕ_E

$$\phi_E = \phi - \left\{\frac{\varepsilon E}{4\pi\varepsilon_0}\right\}^{1/2} \qquad (3.15)$$

where ϕ is the work function as defined in Fig. 7 (ϕ_E and ϕ are expressed in eV).

Let us now compute the number of electrons which evaporate from a unit area of the metal surface in unit time. Assume that the energy L (kinetic) of the electrons inside the metal may be written as a function of the components of momentum and choose the x-axis to be normal to the surface. Then as long as the potential barrier is a function of x-alone the y and z components of the momentum of any electron that passes through the surface are preserved. Hence if a given electron moving towards the surface is to surmount the surface barrier, its energy L must be greater than the barrier height $W_a - \varepsilon\left\{\frac{\varepsilon E}{4\pi\varepsilon_0}\right\}^{1/2}$ by an amount $(p_y^2 + p_z^2)/2m$ (p_i momentum components). Thus

But one has

$$\frac{\partial L}{\partial p_x} dp_x = dL \tag{3.22}$$

when one integrates with respect to dp_x only and the previous integral becomes

$$\nu = \frac{2}{h^3} \int_0^\infty dp_y \int_0^\infty dp_z \int_{L''}^\infty \frac{dL}{e^{\frac{L-L'_0}{kT}} + 1} =$$

$$= -\frac{2kT}{h^3} \int_0^\infty dp_y \int_0^\infty dp_z \int_{L''}^\infty \frac{d e^{\frac{L'_0 - L}{kT}}}{1 + e^{\frac{L'_0 - L}{kT}}}$$

$$= \frac{2kT}{h^3} \int_0^\infty \int_0^\infty \ln\left(1 + e^{\frac{L'_0 - L''}{kT}}\right) dp_y$$

Considering the fact that

$$L'_0 - L'' = -\varepsilon \phi_E - \frac{p_y^2 + p_z^2}{2m}$$

and also that (see p.16)

$$\varepsilon \phi_E >> kT$$

one may write

$$\nu \cong \frac{2kT}{h^3} e^{-\frac{\varepsilon \phi_E}{kT}} \int_0^\infty \int_0^\infty e^{-\frac{p_y^2 + p_z^2}{2mkT}} dp_y\, dp_z =$$

$$= \frac{2kT}{h^3} e^{-\frac{\varepsilon \phi_E}{kT}} \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\frac{\rho^2}{2mkT}} \rho\, d\rho\, d\theta$$

$$= \frac{4\pi m (kT)^2}{h^3} e^{-\frac{\varepsilon \phi_E}{kT}}$$

(3.23)

Thus the current will be

$$I_s = A T^2 e^{-\frac{\varepsilon\phi}{kT} + \varepsilon \cdot \frac{(\varepsilon E)^{1/2}}{kT(4\pi\varepsilon_0)^{1/2}}} \qquad (\phi = eV)$$

$$A = \frac{4\pi m \varepsilon k^2}{h^3} = 120 \frac{amp}{cm^2\, deg^2}$$

If one designates by r the probability that the electrons which have sufficient energy to get over the barrier are reflected back, one finds

$$I_s = A T^2 (1-r) e^{-\frac{\varepsilon\phi}{kT} + \cdots} = A T^2 D e^{-\frac{\varepsilon\phi}{kT} + \frac{\varepsilon\phi'}{kT}}$$

Incidentally D is not independent of T but in the previous formula we merely indicate the way in which it enters into the calculations. Nordheim has shown that $D \sim 1$ for all practical purposes.

The value of the constant A derived on the previous theoretical assumptions is twice as large as the one evaluated experimentally. This deviation was to be expected after all the simplifying assumptions which have been made during the above derivations. Namely:

1. Constant momenta p_y, p_z. This is approximately true because the gradients along the emitting metal surface are not zero.
2. No interaction between the electrons. The assumption that the electronic gas follows Fermi Statistics implies that it is considered as ideal. This, of course, is true to a certain extent at high temperatures.

3. Plane emitting surface. This assumption simplifies the derivations very much because even small surface abnormalities result in appreciable modification of the potential barrier.

4. Work function independent of temperature which is not true completely as one may see from the formulas for the parameter L'--.

CONCLUSION

A bird's-eye view of the phenomenon of thermionic emission has been given in the previous discussion. The emphasis has been put mostly on the Fermi Statistics because it is the author's opinion that in this way all the quantities used in the derivations become meaningful and not arbitrary, to the extent the made assumptions are true. On the other hand Fermi Statistics have been very fruitful in the theory of solids so that their use is completely justified in the case of the phenomenon of evaporation of particles from metal surfaces.

The point of view of escape through a potential barrier is also plausible since it is in perfect agreement with the experimental evidence related both to clean and coated metal surfaces.

A word is due at this point about the actual tendency of modern science to unify the phenomena of the physical world, starting from a few basic assumptions. This is a fortunate situation because otherwise, with the high degree of specialization required on the part of different scientists, it would be reasonable, but not justifiable, to conclude that sooner or later there would be no connection between the different fields, whatsoever.

The attempts of unification seem for the time being complicated and uncomprehensible. This is due to the fact that people are still rather unfamiliar with them or in some other cases that they have been oversimplified to the extent to lose their essential points.

The author does not share the opinion that some of these attempts

do not express anything else than a mathematical or spiritual play on the part of the scientist who suggests them. On the contrary, the author thinks that they stem from an inevitable necessity, which becomes more and more mature as scientific knowledge increases.

Furthermore the author believes that all these suggestions result mostly to constant product rather than ratio of basic assumptionsto knowledge.

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