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ENTROPY. PART II: THERMODYNAMICS AND PERFECT ORDER

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ABSTRACT

Part II of this two-part paper refutes the beliefs about the statistical interpretation of thermodynamics, and the association of entropy with disorder that are summarized in Part I. The refutation of the statistical approach is based on either a nonstatistical unified quantum theory of mechanics and thermodynamics, or an almost equivalent, novel, nonquantal exposition of thermodynamics. Entropy is shown to be: (i) valid for any system (both macroscopic and microscopic, including one-particle systems), and any state (both thermodynamic or stable equilibrium, and not stable equilibrium); (ii) a measure of the quantum-theoretic pliable shape of the molecules of a system; and (iii) a monotonic indicator of order. In contrast to statistics which associates a thermodynamic equilibrium macrostate with the largest number of compatible microstates, the second law avers that, for each set of values of energy, volume, and amounts of constituents of either a macroscopic or a microscopic system, there exists one and only one thermodynamic or stable equilibrium state. So, even if Boltzmann's definition were used, a thermodynamic equilibrium state is one of perfect order.

1 INTRODUCTION

Ever since the unrefined enunciation of the first and second laws of classical thermodynamics by Clausius (1867), the question of the relation between thermodynamics and mechanics has been the subject of intense investigations and controversy. Invariably, Maxwell's (1871) seminal ideas prevail, that is " ... the molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform ...", and "In dealing with masses of matter, while we do not perceive the individual molecules, we are compelled to adopt what I have described as the sta-

tistical method of calculation, ...". Though deeply rooted in classical mechanics, these ideas play a major role in conventional quantal explanations as well.

Statistical theories of thermodynamics yield many correct and practical numerical results about thermodynamic equilibrium states (McQuarrie, 1973; Callen, 1985). Despite these successes, the almost universal efforts to compel thermodynamics to conform to statistical explanations is not justified in the light of many accurate, reproducible, nonstatistical experiences, and is out of step with the responses of the scientific community to the considerations that resulted in modifications of classical mechanics. Examples of such modifications are the theory of relativity, and the theory of quantum mechanics.

Over the past three decades, intrigued and challenged by the prevalent misunderstandings about thermodynamics, a group of us has proposed a resolution of the dilemmas and paradoxes created by statistical explanations. The resolution differs from all statistical interpretations of thermodynamics, in general, and from Maxwell's (1871) explanation, in particular.

The purposes of Part II of this two-part paper are: (i) to summarize a unified quantum theory of mechanics and thermodynamics which leads to the demonstrable conclusion that thermodynamics is a general, nonstatistical theory of physical phenomena; this theory admits mechanics (either classical or conventional quantum) as one limiting case (zero entropy physics), and classical thermodynamics as another limiting case (largest entropy physics); moreover, entropy is a well-defined and mathematically precise monotonic indicator of order; the indicator is a measure of the

quantum-theoretic shape of the constituents of a system, in the same sense that, in classical mechanics, $mv^2/2$ and not some other expression is a measure of the kinetic energy of a particle; (*ii*) to sketch a nonquantal, novel exposition of thermodynamics that is almost equivalent to and results in the same conclusions as the unified theory; and (*iii*) in the light of the unified theory and the new exposition, to evaluate the beliefs summarized and respond to the questions raised in Part I.

The paper is organized as follows. A roadmap through an interwoven and well-documented series of theoretical, nonstatistical developments is discussed in the second section. In the light of these developments, the issues outlined in Part I are clarified in the third section, and conclusions are presented in the fourth section.

2 ROADMAP THROUGH RECENT DEVELOPMENTS

2.1 A unified quantum theory of mechanics and thermodynamics.

Hatsopoulos and Gyftopoulos (1976 a, b, c, d) have proposed a nonstatistical resolution of the dilemmas and paradoxes that have preoccupied generations of physicists over more than a century in their attempts to rationalize the relation between mechanics and thermodynamics. The resolution is based on a unified quantum theory of mechanics and thermodynamics which without modification encompasses all systems (both macroscopic and microscopic, including systems with only one particle), and all states (both thermodynamic or stable equilibrium and not stable equilibrium).

The key for the elimination of all statistical explanations is the discovery that the only quantum-mechanical density operators $\rho > \rho^2$ that are subject to the laws of physics (quantum- theoretic and thermodynamic) are those that can be represented by a homogeneous ensemble (Hatsopoulos and Gyftopoulos, 1976d). In such an ensemble, every member is assigned the same ρ as any other member (Fig. 1) and experimentally (in contrast to algebraically) ρ cannot be decomposed — is unambiguous or irreducible — into a statistical mixture of either projectors or density operators different from ρ. The impossibility of decomposition is analogous to von Neumann's (1955) conclusion that a wave function or projector ($\rho_i = \rho_i^2$) cannot be decomposed into a statistical mixture of states of classical mechanics. Moreover, and perhaps more importantly, the extension of the concept of homogeneity to density operators $\rho > \rho^2$ is accomplished without radical modifications of the quantum-theoretic postulates and theorems about observables, measurement results, values of observables, and
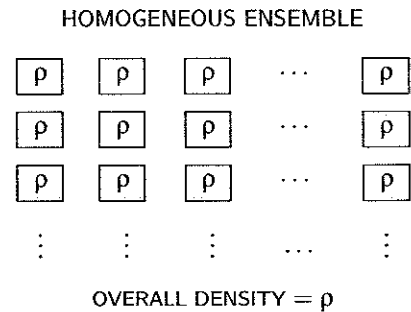

Figure 1. Representation of a homogeneous ensemble.

densities or probabilities of measurement results.

Key concepts — definitions, postulates, and theorems — of the unified theory are discussed also by Gyftopoulos and Çubukçu (1997), and Gyftopoulos (1998a).

2.2 Equation of motion.

Hatsopoulos and Gyftopoulos (1976c) observed that Newton's, Schroedinger's and its equivalent von Neumann's equations of motion are correct but incomplete. More specifically, Schroedinger's equation describes only reversible adiabatic processes that are unitary. However, not all reversible adiabatic processes are unitary, and not all processes are reversible. Moreover, von Neumann's equation (Tolman, 1962) for unitary transformations of ρ in time is not acceptable because both his ρ is a statistical average of wave functions or projectors, and the statistical weight of each wave function is independent of time.

In response to these observations, Hatsopoulos and Gyftopoulos (1976a) postulated a von Neumann-like equation for the unitary evolution in time of unambiguous or irreducible ρ's, and Beretta et al. (1984 and 1985) proposed equations of motion that involve no statistics associated with lack of information or ignorance, or macroscopic justifications, and that account for both all reversible and all irreversible processes.

It is noteworthy that, of all the equations of motion that differ from the Schroedinger equation and that have been proposed in the literature, Kossh and Steffen (1987), and Çubukçu and Gyftopoulos (1995) prove that the equations proposed by Beretta et al. (1984 and 1985) are the only ones that comply with all the criteria that must be satisfied by an equation that accounts for all reversible and all irreversible processes.

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2.3 A novel, nonquantal exposition of thermodynamics.

Gyftopoulos and Beretta (1991a) have composed a novel, nonstatistical exposition in which all concepts of thermodynamics are defined completely and without circular and tautological arguments in terms of only the concepts of space, time, and force or inertial mass. Though the intellectual underpinning is the unified quantum theory cited earlier, the new exposition evolves without any reference to quantum-theoretic concepts, postulates, and theorems. So it can be studied without knowledge of quantum theory.

The order of introduction of concepts, postulates, and theorems is: system (types and amounts of constituents, forces between constituents, and external forces or parameters, such as volume); properties at an instant in time; states; the first law (without the concepts of energy, work, and heat); energy (without work and heat); energy balance; classification of states in terms of time evolutions; thermodynamic or stable equilibrium states; second law (without temperature, heat, and entropy but as an assertion of *existence of one and only one stable equilibrium state for each set of values of energy, volume, and amounts of constituents*); generalized available energy; entropy of any state, stable equilibrium or not, in terms of energy and generalized available energy (and not in terms of temperature and heat); entropy balance; fundamental relation which exists for stable equilibrium states only; temperature, pressure and total potentials in terms of energy, entropy, volume, and amounts of constituents, valid for stable equilibrium states only; the third law; work in terms of energy only; heat in terms of energy, entropy, and temperature only; other interactions; various applications. All concepts and postulates are valid for all systems (both macroscopic and microscopic), and all states (both stable equilibrium and not stable equilibrium).

In this exposition, no statistical considerations whatsoever are required either by any definition or by any postulate or by any theorem.

2.4 Definition of entropy.

The definition of entropy in the novel exposition (Gyftopoulos and Beretta, 1991b; Gyftopoulos and Çubukçu, 1997; and Gyftopoulos, 1998b) differs from and is more general than that in any other exposition of thermodynamics. The new definition: (i) applies to all systems, both macroscopic and microscopic, including one-particle systems; (ii) applies to all states, both stable equilibrium and not stable equilibrium; (iii) most certainly is not statistical; and (iv) satisfies about ten reasonable and readily understandable criteria.
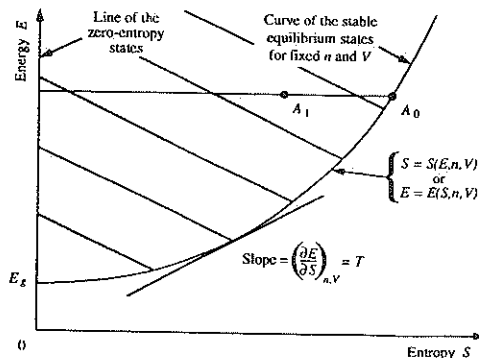

Figure 2. Energy versus entropy graph of a system with fixed values of amounts of constituents and volume, and without upper bound on energy.

2.5 Energy versus entropy graphs.

One result of the novel exposition of import to the purposes of this paper is a graph of energy E versus entropy S (Gyftopoulos and Beretta, 1991c).

At an instant in time, the *state* of a system is defined by the values of the amounts of constituents n, the value of the volume V, and the values of a complete set of independent properties, and can be represented by a point in a multidimensional space with one axis for volume, for each amount of constituent, and for each independent property. Such a representation, however, is unwieldy because the number of independent properties of any system, even a system consisting of one particle only, is infinite. Nevertheless, useful information can be captured by first cutting the multidimensional state space by a hypersurface corresponding to given values of the volume and of each of the amounts of constituents, and then projecting the cut on an energy versus entropy plane. For a system A without upper bound on energy, we prove that the projection must have the shape of the cross-hatched area in Fig. 2.

A point either inside the cross-hatched area or on any line of the surface $S = 0$ represents the projections of an infinite number of states. Each such state has the same values of n, V, E and S but differing values of other properties, and is not a stable or thermodynamic equilibrium state. In particular, the surface $S = 0$ represents all the states encountered in purely mechanical theories of physical phenomena. On the other hand, by virtue of the second law of thermodynamics, each point on the convex curve represents one and only one stable equilibrium state. Said differently, the value of every property, including entropy, is determined solely by the values of E, n, and V, or the value of every property, including energy, is determined solely by the values S, n, and V.

For all classes of states, stable equilibrium or not sta-

ble equilibrium, it is clear that entropy is a property of the constituents of the system and not a statistical measure of ignorance, lack of information, inability to perform detailed calculations, or disorder. The importance of this interpretation of entropy and the conceptual difference of the interpretation from all others in the literature cannot be overemphasized.

2.6 Analytical expression for entropy.

It is self-evident that any explicit analytical expression purporting to represent the concept of entropy must satisfy the criteria resulting from its definition. Gyftopoulos and Çubukçu (1997) prove that none of the expressions existing in the literature, including $k \ln \Omega$, conform with these criteria except for

$$S = -k \operatorname{Tr} [\rho \ln \rho] \qquad (1)$$

provided that ρ is a density operator that cannot be decomposed into other operators, that is, an operator which is represented by a homogeneous ensemble (Fig. 1), where Tr stands for the trace of the operator that follows, and k is Boltzmann's constant.

Though it looks identical to a relation proposed by von Neumann, Eq. (1) differs from von Neumann's relation for a fundamental reason of paramount importance. In the von Neumann relation, ρ is a statistical average of wave functions or projectors. Accordingly, not all members of the ensemble representation of such a ρ are characterized by the same projector. In Eq. (1), ρ is an unambiguous or irreducible operator that can be represented only by a homogeneous ensemble (Fig. 1). If ρ is ambiguous — operationally can be decomposed into a statistical combination of two or more different density operators — Eq. (1) does not represent the entropy of thermodynamics, and neither ρ nor S are subject to the fundamental laws of physics.

2.7 Entropy as a measure of the quantum-theoretic shape of constituents.

In many textbooks (Leighton, 1959; Brandt and Dahmen, 1995), the probability density function of the spatial coordinates is interpreted as the shape of the constituents of a system. An illustration is provided by the wave packet and its evolution in time depicted in Fig. 3. Gyftopoulos (1998 a and b) observed that the same interpretation of the spatial shape can be applied to the probabilities derived from a density operator $\rho > \rho^2$.

If follows that the entropy of thermodynamics (Eq. (1)) is a measure of the quantum-theoretic spatial shape of constituents, in the same sense that $mv^2/2$, and not some
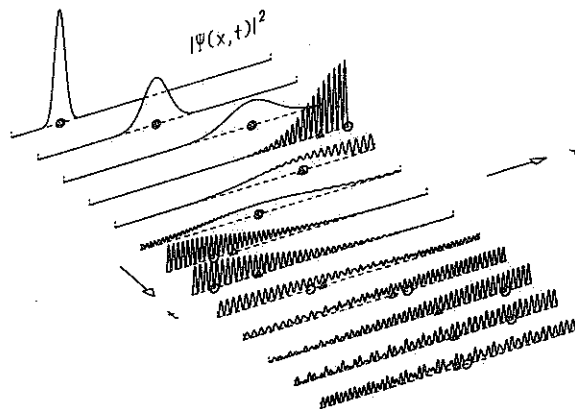

Figure 3. Time evolution of the shape of a wave packet in an infinitely deep, one-dimensional potential well (Brandt and Dahmen, 1995).

other expression, is the measure of the kinetic energy of a molecule.

For given values of the energy E, volume V, and amounts of constituents n, the values of the special measure S range from zero to a largest value. For all density operators that are projectors, $\rho_i = \rho_i^2$, such as the wave packet in Fig. 3, the value of $S = 0$. For all other density operators, $\rho > \rho^2$, $S > 0$ and reaches the largest value for the density operator ρ^0 that corresponds to the unique stable equilibrium state dictated by the second law of thermodynamics for the given values E, V, and n.

With this interpretation in mind, we can think of the spontaneous increase of entropy in the course of irreversible processes as a natural tendency of an atom, molecule, or other system to adapt the shape of its state to the nest of internal and external forces of the system until no further reshaping is possible, that is, for given values of E, V, and n, until the unique ρ^0 and the largest value of S are achieved. The adaptation to the internal and external forces is always in the direction of nondecreasing entropy, and is beautifully exemplified by considering the spontaneous expansion of an amount of colored gas squirted in a corner of a transparent glass container.

2.8 Velocity of any particle of a system in a stable equilibrium state.

For a system in a stable equilibrium state, Gyftopoulos (1998 a and b) proves that the quantum-theoretic value of the velocity of any particle — nucleon, electron, atom, or molecule — equals zero. This zero is the value of the velocity of the particle in question and not an average over velocities of different particles moving in different directions. In other words, if a system is in a thermodynamic or stable equilibrium state, every motion has ceased, nothing moves.

2.9 Entropy of thermodynamics and perfect order.

For a system with given values of E, V, and n, in the unified theory and in the novel exposition we have established the following results: (i) there exists one and only one stable equilibrium state; (ii) in the stable equilibrium state, the entropy has the largest value, and all velocities are null; and (iii) there is an infinite number of states for each value of the entropy smaller than the largest value. In view of these results, we are compelled to conclude that the stable equilibrium state is more orderly than any other state which is turbulent or nonequilibrium, or said differently, that thermodynamic or stable equilibrium represents perfect order rather than disorder. Interestingly enough, because of the uniqueness of the stable equilibrium state for each set of values E, V, and n, the conclusion about order is reached even if we use the definition of disorder proposed by Boltzmann!

This completes the brief description of the roadmap through the interwoven recent developments about thermodynamics and quantum theory, and the definition, analytical expression, and pictorial interpretation of the entropy of thermodynamics. In the next section, we examine the beliefs about statistics and disorder in the light of these developments.

3 REVIEW OF BELIEFS ABOUT STATISTICAL MECHANICS

3.1 Maxwell's views (Part I, Section 2.1, in these proceedings).

Maxwell is correct in asserting that if one accepts the atomic-kinetic viewpoint, entropy and irreversibility cannot be parts of an absolute law of nature. Viewed from that perspective, molecules are hard spheres, and do not have entropy as a private property as they do have inertial mass and energy as private properties. As discussed in Part I, however, the theoretical and empirical evidence with gases under conditions of ideal behavior is contrary to Maxwell's conclusion. To overcome the contradiction, we must abandon the premise that molecules behave as hard spheres and accept the quantum-theoretic idea that molecules have a variable shape. The variable shape is even more elegantly illustrated by our current understanding of the electromagnetic field in a thermodynamic or stable equilibrium state. It is shown (Beretta and Gyftopoulos, 1990) both that each mode of fundamental frequency ν has its own private energy, entropy, and pressure — behaves as each molecule of an ideal gas — and that the integral of each of these three properties over all fundamental frequencies yields the respective calculated and measured property of the black-body. Moreover, and perhaps more importantly, the pliability of the modes to adapt from one volume to another is easier to visualize and comprehend in terms of electromagnetic waves than in terms of molecules. Of course, the quantization of either a particle with nonzero inertial mass or of the electromagnetic field, and the nonclassical explanation of molecular structures and blackbody radiation were not known at the time Maxwell proposed his statistical theory. The contemporary evidence, however, suggests that the visualization of molecules as hard spheres is neither necessary nor sufficient to regularize thermodynamic phenomena. The discussions in Sections 2.4, 2.6, and 2.7 eliminate the classical mechanics premise, and yield results that are consistent with both our understanding of quantum theory, and with all the empirical evidence.

Again, Maxwell is correct in asserting that a being with "fine tactile and perceptive organization" — a demon — could separate fast molecules from slow molecules in a container without performance of work, in violation of the second law of thermodynamics. Here too, his premise is the notion that, at the microscopic level, molecules do not have entropy as a private property. More than 300 publications (Leff and Rex, 1990) discuss the feasibility of the demon by using different specific devices (pistons, photons, ratchets and pawls, and computers), and all conclude that the demon is not feasible because each device of the schemes just cited generates entropy, and at the end of each scheme the entropy change is positive, and so the second law remains intact.

Upon careful study, I find that practically all the publications about the demon are flawed for three fundamental reasons. First, none of the authors of these publications addresses the problem posed by Maxwell, that is, that the demon must accomplish his task without any energetic and/or entropic cost to him whatsoever. In each proposal, the specific devices contribute either energy or entropy, or both and, therefore, do not operate as specified by Maxwell. Second, in each of these publications, the demon is proven infeasible by showing that the entropy increases and so, the authors conclude, the second law is not violated. But this is not correct reasoning. The laws of thermodynamics require that the entropy increase only in irreversible adiabatic processes, but these laws do not require that all adiabatic processes be irreversible. Third, and perhaps most importantly, each of these publications overlooks one of the most elegant and most universal results of thermodynamics, that is, limiting conditions must be and always are established for a reversible process, and not for any particular gizmo for the implementation of the process, because for a given task all reversible processes are equivalent. For example, the Carnot coefficient $(T_1 - T_2)/T_1$ is independent of the engine and its cycle, and an excellent illustration of the remark

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just cited.

One definitive exorcism of the demon (Gyftopoulos, 1998 c) is achieved by an analysis of his demonic act in terms of the graph in Fig. 2. This graph can be regarded as representing the states of the air molecules in the vessel discussed by Maxwell. The air is system A in thermodynamic or stable equilibrium state A_0. The demon is asked to sort the air molecules into swift and slow without any contribution either by him or the environment. If this were possible, the final state of A would be A_1, that is, a state with the same values E, V, and n and less entropy than A_0. But, in the novel exposition of thermodynamics, entropy is not introduced because "we are compelled to adopt the statistical method of calculation" but because it is proven to be a well-defined, nonstatistical, and nondestructible property of each molecule. Accordingly, the demon cannot reduce this molecular entropy without compensation no matter how "fine his tactile and perceptive organization" is. It is clear that this impossibility has nothing to do with either spontaneous entropy generation (irreversibility), or shortcomings of the procedures the demon has at his disposal, or both.

Equivalently, if the demon is regarded as a perpetual motion machine of the second kind (von Smoluchowski, 1914), then his ultimate goal is to extract energy only — make the system do work only — starting from state A_0. But, under Maxwell's specifications, it is clear from the graph in Fig. 2 that inherently each state of energy smaller than E_0 of A_0 has also smaller entropy than S_0. And again, because entropy is a well-defined, nonstatistical, and nondestructible property of each molecule, the demon cannot accomplish his assignment because he is forbidden from compensating for the entropy reduction. If either he were allowed to extract both energy and entropy, or if entropy were not a property of each molecule, the demon's task would be very easy. All he would have to do is cool the air to a temperature smaller that T_0 or extract only energy.

A second definitive exorcism is achieved by an analysis based on the quantum-theoretic results summarized in Section 2.8. To establish the quantum-theoretic value of the velocity of each molecule, the demon must perform measurements on homogeneous ensembles that represent the stable equilibrium state density operator ρ^0. As a result he finds that the value of the velocity of each molecule — not the average of the velocities of many molecules — equals zero. So he concludes that there are no swift and slow molecules to be sorted out. Though he would continue to venerate the imaginative and creative intellect of his father, he regrets that quantum theory was not recognized earlier so that he could have been spared the century-long search for a solution of a problem that does not exist.

3.2 Boltzmann's view (Part I, Section 2.2).

On the occasion of the 150th anniversary of Boltzmann's birthday, a recent essay (Lebowitz, 1993) reminds us of how richly Boltzmann deserves to be admiringly commemorated for the originality and importance of his ideas, discusses the success of Boltzmann's statistical approach in explaining the observed irreversible behavior of macroscopic systems in a manner consistent with their reversible microscopic dynamics, and finds it surprising that there is still so much confusion about the problem of irreversibility.

I could not agree more with the debt we owe to Boltzmann for his trailblazing, stimulating, and fertile contributions to physics. However, both our current theoretical understanding, and empirical evidence suggest that Boltzmann's views about thermodynamics are not indisputable (see Part I, Sections 3 and 4).

In Sections 2.4 and 2.6, we show that entropy is not statistical, and that $k \ln \Omega$ cannot represent the entropy of thermodynamics because it does not satisfy the criteria that must be met by an analytical expression that purports to represent entropy. In Section 2.2, we provide arguments for the need of a complete equation of motion which describes both all reversible processes and irreversible processes, and propose equations that satisfy these criteria. Moreover, Gyftopoulos (1988 c) has shown that the Boltzmann equation that describes the evolution in time of a density function of space and time, even if valid, applies only to very dilute gases, that is, states for which the expectation value of the number of particles is less than unity.

In Section 2.9, we prove that both the novel exposition and the unified theory require that each thermodynamic or stable equilibrium state must be unique because that is the content of the second law of thermodynamics. So, even if we use Boltzmann's criterion of disorder, we must conclude that a thermodynamic equilibrium state represents perfect order.

An empirical result that contradicts the idea that a spontaneous entropy increase implies an increase of disorder is the spontaneous crystallization of an agitated and turbulent fluid. It is hard to argue that an agitated and turbulent fluid is more orderly than a perfect crystal.

Another empirical result is discussed by Bridgman (Denbigh, 1966). He refers to the spontaneous crystallization of a supercooled liquid which results in entropy increase, and for which again is difficult to claim that there has been an increase in disorder. Nevertheless, Denbigh (1966) argues that: " ... though there is a decrease of configurational entropy, consequent on the more orderly arrangement of the lattice as compared to the liquid, there is a more than compensating increase in *thermal energy*, due to the randomization of the liberated potential energy over the vibrational motions of the atoms in the crystal."

There are several objections to Denbigh's insistence to explain the spontaneous entropy increase by invoking a connection between thermodynamic equilibrium and disorder. For example, thermal energy — heat — is not a property of a system. Thermal energy refers to the entities that are exchanged between two systems in the course of a heat interaction. Again, we have proved (Section 2.8) that, in any thermodynamic equilibrium state, the quantum-theoretic value of the velocity of any individual particle equals zero. So there are no vibrational motions in a crystal in a thermodynamic equilibrium state.

In view of these observations, we must conclude that neither Boltzmann's expression for the entropy of thermodynamics, nor his association of entropy with disorder are justified either by quantum theory or by the empirical evidence.

3.3 Brillouin's view (Part I, Section 2.3).

Brillouin and other scientists (Jaynes, 1957 a and b; Katz, 1967) propose a statistical interpretation of thermodynamics, and an informational explanation of entropy. They argue that the better informed an observer is about the state of a system, the lesser the entropy that he assigns to that state. In the light of both the unified theory, and the empirical evidence, both the informational interpretation of thermodynamics, and the informational explanation of entropy cannot be correct. Given a system, its state is uniquely determined by a density operator ρ or by a complete set of values of independent properties. Moreover, the performance of the system is dictated exclusively by the values of the concepts just cited and not by the degree of information of an observer. For example, if the system is in a stable equilibrium state, the operator is ρ^0, the entropy is the largest of the entropies of all the states that share the same values E, V, and n, and no amount of additional information affects either ρ^0 or $(-k \operatorname{Tr} \rho^0 \ln \rho^0)$. Again, let us consider two identical electricity storage batteries, each having the same value of energy but one being fully charged and the other fully discharged internally and spontaneously. A universal experience is that every observer, who knows nothing about thermodynamics, information theory, entropy, and batteries, will find that one battery powers a flashlight, whereas the other does not. Similar comments can be made about a myriad of other systems in either a thermodynamic equilibrium state or not in such a state. So the statistical interpretation of thermodynamics, and the concomitant informational explanation of entropy cannot be correct.

In addition, Brillouin associates lack of information, entropy, and disorder with lack of specificity of the positions and velocities of the molecules of an ideal gas. As we discussed in Part I, Sections 3 and 4, however, this association leads to inconsistencies.

3.4 Feynmann's views (Part I, Section 2.4).

Feynmann wonders how we get irreversibility out of the equations of mechanics which describe only reversible phenomena, and associates entropy with disorder.

Our answer to Feynmann's perceptive comments is that the equations of motion of mechanics are correct but incomplete, and that an appropriate proposal for their completion has been made (Section 2.2). Unfortunately, because most scientists believe that all questions about thermodynamics have been answered by statistical mechanics, very few individuals recognize that there remain important questions to be answered, and that the discovery of the complete equation of motion of physics is one of them.

Our position about disorder is discussed in Section 3.2.

3.5 Penrose's views (Part I, Section 2.5).

Penrose as well as many other eminent physicists associate the increase of entropy with the arrow of time. Despite the almost universal agreement about this association, I believe that its validity is not correct, and will discuss it in a future communication.

Next, Penrose asserts that entropy could not be a "very clear-cut scientific concept." I could not disagree more with this assertion because, in the novel exposition, the definition of entropy is as explicit, complete, noncircular, and precise as that of any other important concept of physics (Section 2.4).

Next, Penrose questions the importance of conceiving concepts, such as irreversibility , that are "practical." I have two responses to this kind of questioning. First, thermodynamic thinking plays a very important role in cosmology. Second, our ability to control entropy generation — to use processes that are as close to reversible as we can afford — is of paramount importance to the protection of our environment, and the wise use of our limited resources of energy and materials. I would be very surprised if there were many scientists who feel that physics should not be concerned with the well-being of humanity.

Finally, Penrose claims that entropy does not have an analytical expression as do energy, momentum, and angular momentum. The discussions in Sections 2.6 and 2.7, and many relations of classical thermodynamics contradict this claim, and provide incontrovertible evidence that the concept of entropy not only has an analytical expression but it is as well founded as any other well-defined concept of physics including energy, momentum, and angular momentum.

4 CONCLUSIONS

The brief summaries and the references listed in Section 2, provide a lot of information for study. The main message is that thermodynamics is a well-defined, nonstatistical general theory that includes mechanics and conventional quantum mechanics as special cases.

The conclusions in Section 3 contradict the beliefs about thermodynamics, statistical mechanics, and entropy discussed in Part I, and are consistent with the empirical evidence. In particular, entropy is shown to be a measure of the quantum-theoretic spatial shape of the molecules of a system. For given values of E, V, and n, the value of this measure increases from zero to a maximum, and the latter value corresponds to a unique state which is the most ordered of all the states that have the same values E, V, and n.

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