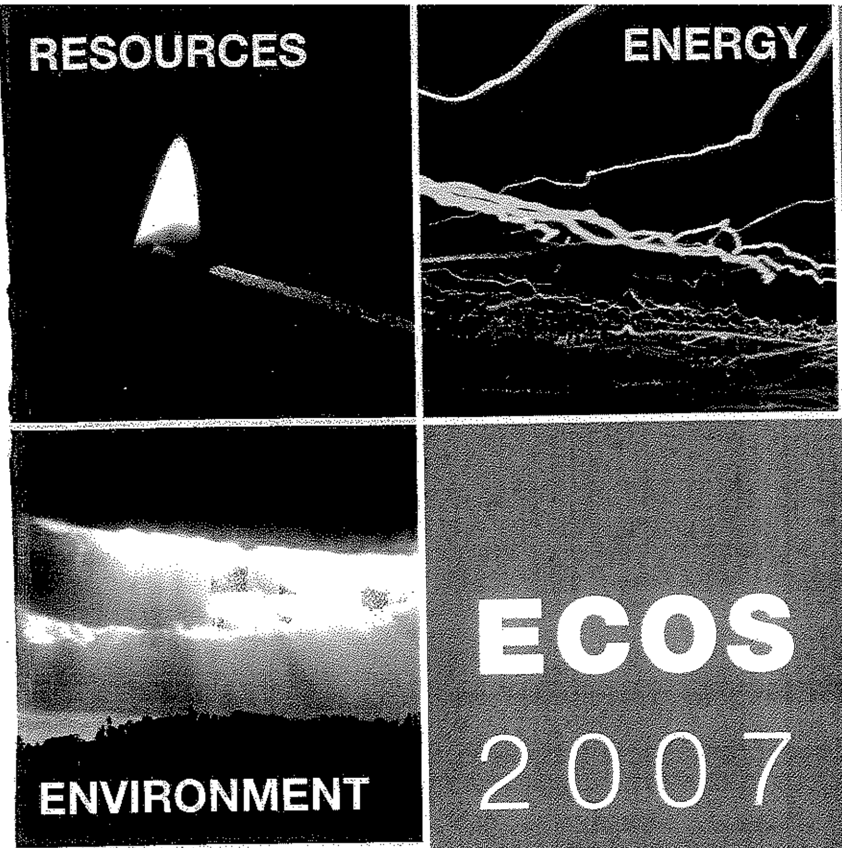

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KEYNOTES

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Quantum Paradoxology and Quantum Computation

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ABSTRACT: We review both the Einstein, Podolsky, Rosen (EPR) paper, and Schrödinger's responses, and find that both are not consistent with the current understanding of quantum theory and thermodynamics. Because both contributions play a leading role in discussions of the fascinating and promising fields of quantum computation and quantum information, we hope our review will be helpful to researchers in these fields. A summary of our understanding of quantum theory and thermodynamics is presented in the Appendix. In addition to its role in the discussion of quantum computation, the summary provides tools for analyses of all systems (large or small) in any state (unsteady, steady, nonequilibrium, equilibrium, and stable equilibrium). As such, it may stimulate new interests of the attendees of this Conference in nanotechnology and other emerging fields.

Key concepts: Nonstatistical quantum thermodynamics, expansion versus superposition of wave functions and/or projectors, nonstatistical density operators, homogeneous ensembles, state, uncertainty relations, equation of motion of quantum thermodynamics, entanglement, measurement result.

NOMENCLATURE

A, B, …	operators representing properties
a_i, b_i, …	ith eigenvalues of operators A, B, … respectively
ΔA, Δp_x, Δx	standard deviations of ensembles of measurement results of properties represented by operators A, p_x, x respectively
g	degeneracy of an eigenvalue of a given operator
h	Planck's constant
$\hbar$	Planck's constant divided by 2π
$\mathcal{H}$	Hilbert space
p_x	momentum operator along x direction

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U	von Neumann's statistical density operator (Schrödinger's notation)		*Greek letters*	
t	time coordinate		ρ	statistical or nonstatistical density operator
T	time interval		τ	time interval
x	space coordinate along the x direction		φ	wave function
$\lvert x\rangle$	ket, Dirac's notation of a wave function		ψ, Ψ	wave functions
$\langle x\rangle$	value of observable property x		ρ_i	projector $\left(\rho_i = \rho_i^2\right)$, or statistical operator $\left(\rho_i > \rho_i^2\right)$

1. INTRODUCTION

The great and very important interest in quantum computation and quantum information[1] behooves us to review the current definitions, postulates, and major theorems of quantum theory to see whether they are correctly and consistently used by researchers in the development of the fascinating and promising field of quantum computers. We notice that practically all discussions of quantum computers and quantum information involve: (i) the paradoxology of the famous EPR paper[2]; (ii) Schrödinger's cat paradox and his responses to the EPR paper[3-6]; and the concept of entanglement.

In this essay, we review the issues raised in the EPR paper regarding the completeness of quantum theory, Schrödinger's responses to the issues just cited, and the concept of entanglement, and regret to report that they misrepresent the definitions, postulates, and principal theorems of quantum theory. The misrepresentations arise from: lack of clear and/or complete definitions at an instant in time of the concepts system, property, and state; use of a postulate that has been proven to be false; and misinterpretation of an expansion of a wave function in terms of a complete set of orthonormal eigenfunctions as a superposition of the eigenfunctions.

The basis of our remarks is a revolutionary conception of quantum theory and thermodynamics without probabilities of statistical mechanics. A brief summary is presented in Appendix A.

2. CAN QUANTUM MECHANICAL DESCRIPTION OF PHYSICAL REALITY BE COMPLETE?

In the abstract of the EPR paper, the authors aver: "In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by noncommuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete."

The conclusion about a wave function is both false and correct. As we discuss in the Appendix (A V), the description of the probabilities of the physical reality represented by a wave function or projector $\rho_i = \rho_i^2$ is complete for phenomena that correspond to zero entropy physics and, therefore, the sweeping conclusion just cited is not correct. On the other hand, probabilities associated with measurement results may require a representation by a density operator $\rho > \rho^2$ (A III, A V) that involves no statistics of the type introduced in statistical quantum mechanics. In sharp contrast to the density operator defined in statistical theories of physics, the density operator ρ involves only quantum probabilities, and is represented solely by a homogeneous ensemble, that is, an ensemble of identical systems, identically prepared in which *each member is characterized by the same density operator* $\rho > \rho^2$ *as the ensemble, and corresponds to nonzero entropy physics*. Therefore, the conclusion reached by EPR is correct but not for the reason cited in their paper.

It is noteworthy that the concept of a homogeneous ensemble was introduced by von Neumann[7] only for wave functions or projectors. The revolutionary (in the sense of Kuhn[8]) recognition that the concept applies also to density operators that involve no probabilities of statistical physics was recognized by Hatsopoulos and Gyftopoulos[9], and Jauch[10], and observed by Schrödinger[6] who however did not make use of his observation. For further discussion see Dirac's presentation in Section 3.

The concept of entropy referred to in the preceding comments differs from all the concepts introduced in textbooks and scientific articles on quantum mechanics, thermodynamics, and statistical physics. The new concept is shown to be both a nonstatistical intrinsic property of any system (both macroscopic and microscopic, including one spin) in any state (both thermodynamic equilibrium and not thermodynamic equilibrium), and a measure of either the quantum-mechanical spatial shape of the constituents of the system[11,12], or the orientation of spins within or on the Block sphere.

Next, we consider the EPR statement that "In quantum mechanics in the case of two physical quantities described by noncommuting operators, the knowledge of one precludes the knowledge of the other".

This statement is not correct. The proof is given in A XI and A XIV where we show that: (i) the value of an observable represented by an operator A is an expectation value deter-mined by an ensemble of measurement results and not by the result of a single measurement; and (ii) the expectation value $\langle A \rangle$ is independent of that of any other observable represented by an operator B regardless of whether AB-BA=0 or $\neq 0$.

EPR conclude that the description of reality as given by a wave function is not complete. They say: "Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. ... We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*"

The statements just cited are excellent and consistent with the thoughts of many preeminent physicists, including Margenau[13].

Next EPR assert: "To illustrate the ideas involved let us consider the quantum-mechanical description of the behavior of a particle having a single degree of freedom. The fundamental concept of the theory is the concept of *state*, which is supposed to be completely characterized by the wave function ψ, which is a function of the variables chosen to describe the particle's behavior. Corresponding to each physically observable quantity A there is an operator, which may be designated by the same letter."

We find this assertion misconceived because a wave function does not determine the state. The definition of state requires the definitions of both a system (A I) as an entity separable from and uncorrelated with its environment, and the values of a complete set of linearly independent properties (A VI, A VII, and A XIV).

Next, EPR consider a particle described by a momentum eigenfunction, and conclude that "such a particle has momentum but *when the momentum of a particle is known, its coordinate has no physical reality.*"

The conclusion just cited is not correct for the following reasons: (i) If a momentum *measurement yields the value* p_0, that result does not necessarily mean the system immediately after the measurement is in a state for which the probabilities are described by the momentum eigenfunction corresponding to p_0 (A X to A XIV); (ii) The EPR conclusion is based on the so-called von Neumann projection or collapse of the wave function postulate which is proven to be invalid[14-17] (A XVI); and (iii) If the expectation value of momentum measurement results is the p_0, then this does not mean that the coordinate of the particle has no physical meaning. It means that the standard deviation of momentum measurement results $\Delta p = 0$, and the standard deviation of position measurement results $\Delta x = \infty$ so that $\Delta x \Delta p = \infty \times 0 \geq \hbar/2$. On the other hand, if the particle is confined within a one-dimensional infinitely deep potential well of width *L*, then the standard deviation of position measurement results $\Delta x < L$, and the wave function is not and cannot be a momentum eigenfunction because, if it were, then $\Delta p = 0$ and the uncertainty relation is violated, that is, $\Delta x \Delta p < L \times 0 = 0$ and not $\geq \hbar/2$.

Next, EPR aver: "In quantum mechanics it is usually assumed that the wave function *does* contain a complete description of the physical reality of the system in the state to which it corresponds. At first sight this assumption is entirely reasonable, for the information obtainable from a wave function seems to correspond exactly to what can be measured without altering the state of the system. We shall show, however, that this assumption, together with the criterion of reality given above, leads to a contradiction." "For this purpose let us suppose that we have two systems, I and II, which we permit to interact from the time $t = 0$ to $t = T$, after which time we suppose that there is no longer any interaction between the two parts. We suppose further that the states of the two systems before $t = 0$ were known. We can then calculate with the help of Schrödinger's equation the state of the combined system I + II at any subsequent time $t > T$. ... We cannot, however, calculate the state in which either one of the two systems is left after the interaction. This, according to quantum mechanics, can be done only with the help of further measurements, by a process known as the *reduction of the wave packet.*"

After a detailed analysis, EPR conclude: "We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. ... Thus, *it is possible to assign two different wave functions to the same reality* (the second system after the interaction with the first)."

In general, the calculations for $t > T$ are not correct for several reasons: (i) For example, assume that for $t \leq 0$ system I is a proton in a box, and system II an electron in a box, and that during the interaction from $t = 0$ to $t = T$ the two particles combine and form a hydrogen atom. The Hamiltonian operator after the interaction includes the potential energy between the proton and the electron, that is, a term absent from the Hamiltonian operator of the proton, and the Hamiltonian operator of the electron. As a result, no two systems can be identified after the interaction; (ii) As we discuss earlier, the reduction of the wave packet has been proven to be invalid. Accordingly, the conclusion that two different wave functions can be assigned to the same reality is not valid; and (iii) After the interaction is over, the two parts may be separable but correlated, that is "entangled"[4]. If this is the case, defining two systems after the interaction amounts to neglect of correlations, and results in an increase of the non-statistical entropy. But such an increase cannot be accounted either by a wave function Ψ or by its unitary evolution in time dictated by the Schrödinger equation of motion. Because every system in any state has entropy as a fundamental and intrinsic property of the constituents (A V), entropy should not be created and/or destroyed by inappropriate mathematical representations.

3. THE PRESENT SITUATION IN QUANTUM MECHANICS

Schrödinger authored three articles in German[3] that have been translated into English[4]. These articles include a one paragraph description of the "cat paradox". In addition, two presentations were made on Schrödinger's behalf at the Cambridge Philosophical Society by Born[5], and Dirac[6]. In both presentations Schrödinger acknowledges that the EPR paper motivated his offerings. He begins his discussion with the following statements:[4] "*Statistics of Model Variables in Quantum Mechanics*. At the pivot point of contemporary quantum mechanics (Q.M.) stands a doctrine, that perhaps may yet undergo many shifts of meaning but that will not, I am convinced, cease to be the pivot point. It is this, that models with determining parts that uniquely determine each other, as do the classical ones, cannot do justice in nature. One might think that for anyone believing this, the classical models have played out their roles. But this is not the case. *A*. The classical concept of state becomes lost, in that at most a well-chosen *half* of a complete set of variables can be assigned definite numerical values; ... The other half then remains completely indeterminate, while supernumerary parts can show highly varying degrees of indeterminacy. In general, of a complete set ... *all* will be known only uncertainly. One can best keep track of the degree of uncertainty by following classical mechanics and choosing variables arranged *in pairs* of so-called canonically-conjugate ones. The simplest example is a space coordinate x of a point mass and the component p_x along the same direction, its linear momentum (i.e, mass times velocity). Two such constrain each other in the precision with which they may be simultaneously known, in that the product of their tolerance – or variation-widths (customarily designated by putting a Δ ahead of the quantity) cannot fall *below* the magnitude of a certain universal constant, thus $\Delta x \cdot \Delta p_x \geq (\text{Planck's constant})/2\pi = \hbar$".

The conclusions just cited are not correct because they overlook both the universal definition of state (A XIV), and the meaning of uncertainty relations[10] (A XV).

Next, Schrödinger states: "*B*. If even at any given moment not all variables are determined by some of them, then of course neither are they all determined for a later moment by data obtainable earlier. This may be called a break with causality, but in view of *A*. it is no-thing essentially new. If a classical state does not exist at any moment, it can hardly change causally. What do change are the *statistics* or *probabilities*, these moreover causally. Individual variables meanwhile may become more, or less, uncertain. Overall it may be said that the total precision of the description does not change with time, because the principle of limitations described under *A*. remains the same at every moment."

These remarks are not consistent with either the equation of motion of quantum theory, or the definition of state. What defines the state at an instant in time is a set of expectation values (A XIV). If we restrict our considerations to probabilities that are described by a projector, then for any expectation value $\langle \mathrm{F} \rangle$ it is readily shown that[18]

$$\frac{d\langle \mathrm{F} \rangle}{dt} = \frac{\mathrm{i}}{\hbar}\langle \mathrm{HF} - \mathrm{FH} \rangle + \frac{\partial \langle \mathrm{F} \rangle}{\partial t}$$

where H is the Hamiltonian operator of the system.

Next, Schrödinger raises the question "Can one base the theory on ideal ensembles?", and responds as follows: "The classical model plays a Protean role in Q.M. Each of its determining parts can under certain circumstances become an object of interest and achieve a certain reality. But never all of them together – now it is these, now those, and indeed always at most *half* of the complete set of variables al-lowed by a full picture of the momentary state. Meantime, how about the others? Have they then no reality, perhaps (pardon the expression) a blurred reality; or are all of them always real and is it merely, according to Theorem *A*. that simultaneous *knowledge* of them is ruled out?"

As we discuss earlier, this dilemma does not exist if the proper interpretation of quantum theory is followed.

Next, Schrödinger elaborates on the issue of blurred variables, and introduces his cat paradox. Upon using the definitions, postulates, and major theorems of quantum theory (see Appendix), we conclude that there is no "cat paradox". To facilitate our discussion, we use a cartoon[19] (Figure 1) that correctly claims to represent the cat paradox. The cartoon shows a ket that presumably can be represented by a superposition (and not by an expansion) in terms of two kets, one consisting of a radio-activity source and a live cat at an instant in time *t*, and the other a radioactivity source that has decayed at time $t+\tau$ and a dead cat that has been poisoned by the release of hydro-cyanic acid induced by the radiation emitted at $t+\tau$.

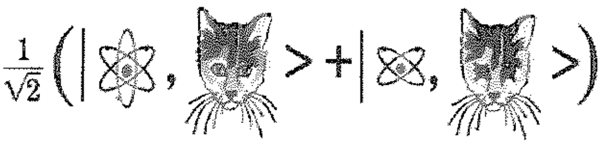

Figure 1: Cartoon representing Schrödinger's cat paradox. (Reprinted from Physics Today by permission of the authors and Physics Today.)

Such a superposition corresponds to no physical reality because a ket is valid at a specific instant in time, and therefore cannot be represented by an expansion, let alone a super-position, in terms of two kets, one of which applies at time t, and the other at time $t+\tau$. Moreover, and for sure more importantly, a radioactivity source prior to decay is a system, that is, an entity both separable from and uncorrelated with its environment which includes a live cat and its life support interactions, such as breathing, drinking, eating, and (excuse the expression) other necessities of living beings. Solid and incontrovertible evidence for the physical reality just cited is provided by a very large number of radioactivity sources devoid of evil contraptions in hospitals, science and engineering laboratories, and nuclear energy installations, and a myriad of creatures, including human beings and cats, that live happily around these sources.

For clarity and avoidance of misinterpretations, at time t the Hilbert space of the radioactivity source and the cat must be the direct product $\mathcal{H}_{r1} \otimes \mathcal{H}_{lc}$, and the catalog of probabilities by the direct product $\rho_{r1} \otimes \rho_{lc}$, where $\mathcal{H}_{i}$ and ρ_i for $i = r1, lc$ are the Hilbert spaces and probability catalogs of the radioactivity source and the live cat, respectively. Most likely, even though not necessarily, the probability catalogs are density operators for both the radioactivity source and the live cat and not projectors or, equivalently, the entropy of each of these two systems is not zero. Moreover, each of the probability catalogs ρ_{r1} and ρ_{lc} is represented by a homogeneous or irreducible ensemble of identical systems, identically prepared (A III and A V). It is clear that in the time interval t to $t+\tau$ the quantum representation of the radioactivity source and the live cat does not involve a radioactivity source that has decayed and a dead cat.

At $t+\tau$, however, if radiation is emitted from the source and precipitates the poisoning and death of the cat, then we have an entirely new situation, that is, two new systems. The source is a system with fewer radioactive nuclei than were present at time t, and the cat is an entirely new system because a dead cat does not need to and does not interact with any life support systems. At this time, the Hilbert space for the two systems is $\mathcal{H}_{r2} \otimes \mathcal{H}_{dc}$, and the probability catalog is $\rho_{r2} \otimes \rho_{dc}$, where the subscript r2 denotes the radioactivity source with fewer radioactive nuclei than at time t, and dc the dead cat. It is clear that neither the new systems nor their probability catalogs have any part that refers to time t. As a result, no inference can be made about the radioactivity source prior to decay and the live cat by studying the radioactivity source after the decay and the dead cat because in each of the two intervals t to $t+\tau$ and (time) $>t+\tau$ each of the two entities is both separable from and uncorrelated with its environment, and therefore can be identified as a system.

After a discussion of theories of measurement, Schrödinger considers two systems that interact with each other for a certain time and then are separated. The following are some of his statements: "This is the point. Whenever one has a complete expectation–catalog – a maximum total knowledge – a ψ-function – for two completely separated bodies, or, in better terms, for each of them singly, then one obviously has it also for the two bodies together, i.e., if one imagines that neither of them singly but rather the two of them together make up the object of interest, of our questions about the future. But the converse is not true. *Maximal knowledge of a total system does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all.* Thus it may be that some part of what one knows may pertain to relations or stipulations between the two sub-systems (we shall limit ourselves to two), as follows: if a particular measurement on the first system yields *this* result, then for a particular measurement on the second the valid expectation statistics are such and such; but if the measurement in question on the first system should have *that* result, then some other expectation holds for that on the second; should a third result occur for the first, then still another expectation applies to the second; ... In this way, any measurement process at all or, what amounts to the same, any variable at all of the second system can be tied to the not-yet-known value of any variable at all of the first, and of course *vice versa* also. If that is the case, if such conditional statements occur in the combined catalog, *then it can not possibly be the maximal in regard to the individual systems.* ... That a portion of the knowledge should float in the form of disjunctive conditional statements *between* the two systems can certainly not happen if we bring up the two from opposite ends of the world and juxtapose them without interaction. For then indeed the two "know" nothing about each other. A measurement on one cannot possibly furnish any grasp of what is to be expected of the other. Any "entanglement of predictions" that takes place can obviously only go back to the fact that the two bodies at some earlier time formed in a true sense *one* system, that is were interacting, and have left behind *traces* on each other."

Earlier we discuss the points raised in the preceding statements except for entanglement which we discuss later. We will see that, as a result of entanglement, no two systems can be defined after the interactions because the two parts may be separable from but correlated with each other.

4. PROBABILITY RELATIONS BETWEEN TWO SYSTEMS

Two communications were presented on Schrödinger's behalf at the Cambridge Philosophical Society by Born[5] and Dirac[6]. In both communications Schrödinger provides detailed mathematical relations that presumably describe what happens to two systems both before and after a temporary interaction.

In the communication presented by Born, Schrödinger says:

1. "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one privilege; to restrict the experiments to one only of the two systems. After re-establishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement.* Its sinister importance is due to its being involved in every measuring process and therefore forming the basis of the quantum theory of measurement, threatening us thereby with at least a *regressus in infinitum*, since it will be noticed that the procedure itself involves measurement."

As we discuss earlier, in quantum theory the definition of a system requires that it be separable from and uncorrelated with its environment (A I). In principle, separability and lack of correlations are subject to experimental verification. For example, if the probabilities of the whole are found to be described by $\psi(x_1, x_2)$, and the probabilities of the two parts by $\varphi_1(x_1)$ and $\varphi_2(x_2)$, respectively, then two systems are separable and identifiable if and only if $\psi(x_1, x_2) = \varphi_1(x_1)\varphi_2(x_2)$, and neither separable nor identifiable if $\psi(x_1, x_2) \neq \varphi_1(x_1)\varphi_2(x_2)$.

2. "Attention has recently been called[2] to the obvious but very disconcerting fact that even though we restrict the disentangling measurements to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are *entirely* arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it. This paper does not

aim at a solution of the paradox, it rather adds to it, if possible. A hint as regards that presumed obstacle will be found at the end."

Earlier we show that there exists neither an EPR nor a cat paradox. In particular, the cat paradox is disproven because it is based on the misconception that a probability catalog – projector or density operator – can be represented by a superposition of two probability catalogs, each of which applies at a different instant in time. Such a misconception is contrary to the structure of all non-relativistic paradigms of physics because in each of these paradigms the definitions of the concepts system, property, and state refer to one instant in time, and the evolution in time is accounted by the equation of motion of each paradigm.

The hint alluded to earlier states: "The paradox would be shaken, though, if an observation did not *relate* to a definite moment. But this would make the present interpretation of quantum mechanics meaningless, because at present the *objects* of its predictions are considered to be the results of measurements for definite moments of time."

The hint is excellent, and proves that there is no paradox.

In the introductory remarks of the Dirac communication[6], Schrödinger says: "An earlier paper[5] dealt with the following fact. If for a system which consists of two entirely separated systems the representative (or wave function) is known, then the current interpretation of quantum mechanics obliges us to admit *not only* that by suitable measurements, taken on *one* of the two parts only, the state (or representative or wave function) of the *other* part can be deter-mined without interfering with it, *but also* that, in spite of this noninterference, the state arrived at *depends* quite decidedly on *what* measurements one chooses to take – not only on the results they yield. ... For it will be shown that *in general* a sophisticated experimenter can, by a suitable device which does *not* involve measuring noncommuting variables, produce a nonvanishing probability of driving the system into any state he chooses; ... The statement is hardly more than a corollary to a theorem about "mixtures"[7]..."

After detailed mathematical analyses of a system for which the probabilities are described by a statistical von Neumann density operator U, Schrödinger concludes that: (i) the expectation value $\langle A \rangle$ of an observable A is given by the relation $\langle A \rangle = \mathrm{Tr}[AU]$; and (ii) "Since the mean values are all that quantum mechanics predicts at all, the knowledge of U in a definite frame of reference *exhausts* our real knowledge of the situation, just as in the case of a "state" the wave function exhausts it. ... U is von Neumann's Statistical Operator. Its matrix is hermitian. It has the formal character of a real physical variable, but the physical meaning of a wave function, that is to say it describes the instantaneous physical situation of the system."

Earlier we found that an experimenter cannot determine the wave function of a system without performing direct measurements on it. We return to the issue here because of the association that Schrödinger makes between expectation values and a density operator U, defined as a statistical average of wave functions or projectors. To be sure, this is contrary to our proven result that density operators in quantum theory are defined exclusively by quantum mechanical probabilities, and not by quantum mechanical probabilities represented by projectors, and statistical probabilities of statistical theories of physics. However, Schrödinger's discussion of U contradicts his definition, and makes U a density operator that is irreducible or unambiguous (A V). Specifically, he emphasizes that "U exhausts our real knowledge of the situation" and therefore that "there is no way of partitioning U into a statistical average of projectors – wave functions". Moreover, he asserts that "U is determined by the only measurable quantities, that is the expectation values."

All these remarks are correct and consistent with the exposition of quantum theory in the Appendix, and not with von Neumann's statistical interpretation of density operators (A XIV). In essence, Schrödinger discovered the density operators of the unified quantum theory of mechanics and thermodynamics but did not pursue the consequences of his discovery.

5. QUANTUM ENTANGLEMENT: A MODERN PERSPECTIVE

The title of this section is identical to that of a recent article[19]. We discuss it because it reflects several misconceptions that are present in many of the publications on quantum computation and quantum information. The article begins with brief discussions of the EPR paper and Schrödinger's cat paradox, illustrated by the cartoon shown in Figure 1, and then concentrates on the concept of "entanglement in a quantum system".

Paradoxes: Earlier we show that there are no EPR and Schrödinger's cat paradoxes.

Entanglement: This concept is introduced by an allegory. "An experimentalist, Alice, wishes to send an unknown state $|\mathrm{s}\rangle = \alpha\,|\,0\rangle + \beta\,|\,1\rangle$ of a two level quantum system to another experimentalist, Bob, in a distant laboratory. ... Alice and Bob do not have the means of directly transmitting the quantum system from one place to another ... but let us imagine that they do share an entangled state. Consider the case in which Alice and Bob each have one spin of a shared singlet state of two spin-1/2 particles $|\Psi^-\rangle = (|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)/\sqrt{2}$, also called EPR pair. Alice can transmit her spin $|\mathrm{s}\rangle$ to Bob by performing a certain joint measurement on her spin $|\mathrm{s}\rangle$ and her half of the EPR pair. She tells Bob the result of her measurement and depending on her information, Bob rotates his half of the EPR pair to obtain the state $|\mathrm{s}\rangle$."

There are three statements in the pre-ceding quote contrary to quantum theory: (i) In principle, an infinite number of measurements on an ensemble of identical systems, identically prepared is needed to determine $|s>$, regardless of the values of α and $\beta\left(\alpha^2+\beta^2=1\right)$. So how can one measurement reveal any information about $|s\rangle$?; (ii) The ket $|\Psi^-\rangle$ is not an EPR state. It is an expansion of $|\Psi^-\rangle$ (*not a superposition*) in terms of two orthonormal eigenkets of a two-spin system; and (iii) a system is not a state, and a state is not a system.

Next, the authors[19] say: "The spin-singlet EPR state that Alice and Bob share in quantum teleportation is called a maximally entangled state. Even though the two spins together constitute a definite pure state, each spin state is maximally undetermined or mixed when considered separately. In mathematical terms, Alice's local density matrix – obtained by tracing over Bob's spin degrees of freedom, $\mathrm{Tr}_B\left(|\Psi^-\rangle\langle\Psi^-|\right)$ – has equal probability for spin up and spin down. In keeping with Schrödinger's understanding of entanglement, one measures the amount of entanglement in a general pure state φ in terms of the lack of information about its local parts. The von Neumann entropy $S(\rho)=-\mathrm{Tr}(\rho\log\rho)$ is used as a measure of that information. In other words, the entropy of entanglement E of the pure state φ is equal to the von Neumann entropy of, say, Alice's density matrix $\rho=\mathrm{Tr}_B|\varphi\rangle\langle\varphi|$."

The statements just cited misrepresent the theory of quantum phenomena. The *thermodynamic entropy* of any projector $|\Psi^-\rangle\langle\Psi^-|$ or $|\varphi\rangle\langle\varphi|$ is equal to zero. The von Neumann entropy is not relevant to this discussion, let alone the fact that it does not represent the entropy of thermodynamics (A V).

6. CONCLUSIONS

Upon detailed and close scrutiny of the ideas presented in the EPR paper and in Schrödinger's responses, we find many faulty conclusions about the very successful theories of quantum mechanics and thermodynamics. We show that the faulty conclusions are due to lack of correct definitions of basic concepts, use of a postulate that is proven not to be valid, and misinterpretations of key theorems of the theory. We also find that the faulty conclusions have permeated the theoretical underpinnings of quantum computation and quantum information. Our criteria are based on an exposition of quantum theory that unifies quantum and thermodynamic ideas without resort to statistical probabilities, and that is summarized in the following Appendix.

APPENDIX: QUANTUM THEORY

We present a summary of nonrelativistic quantum theory that differs from the presentations in practically every textbook on the subject. The key differences are the discoveries that for a broad class of quantum-mechanical problems: (i) the probabilities associated with ensembles of measurement results at an instant in time require a mathematical concept delimited by but more general than a wave function or projector; and (ii) the evolution in time of the new mathematical concept requires an equation of motion delimited by but more general than the Schrödinger equation. Our definitions, postulates, and major theorems of quantum theory are based on statements made by Park and Margenau[20].

The first difference is the recognition[9] that there exist two classes of quantum problems: (i) those that correspond to probabilities described by a wave function or projector; and (ii) those that correspond to probabilities described by a density operator which is not a statistical average of projectors, i.e., not a combination of quantum and statistical probabilities. The second difference is the recognition that the evolution in time of nonstatistical density operators requires a nonlinear equation of motion, and the discovery of such an equation by Beretta et al[21,22].

KINEMATICS: DEFINITIONS, POSTULATES, AND THEOREMS AT AN INSTANT IN TIME.

A I: *System.* The term *system* means a set of specified types and amounts of constituents, confined and controlled by a nest of internal and external forces. Internal forces arise from interactions between constituents.

External forces arise from conditions imposed by force fields outside the system. The effect of each field depends only on the spatial coordinates of each constituent and not on any coordinates of constituents that are not included in the system, that is, the system is *separable* from its environment. In addition, in order to be totally independent and fully identifiable, the system must be *statistically uncorrelated* with its environment.

A II: *System postulate.* To every system there corresponds a complex, separate, complete, inner product space, a Hilbert space $\underline{\mathcal{H}}$. The Hilbert space of a composite system of two distinguishable and identifiable subsystems 1 and 2 is the direct product space $\underline{\mathcal{H}}^1 \otimes \underline{\mathcal{H}}^2$.

A III: *Homogeneous or unambiguous ensemble.* At an instant in time, an ensemble of identical systems is called *homogeneous or unambiguous* if and only if upon subdivision into subensembles in any conceivable way that does not perturb any member, each subensemble yields in every respect measurement results identical to the corresponding results obtained from the ensemble. Other criteria have also been defined[9].

A IV: *Preparation.* A *preparation* is a reproducible scheme used to generate one or more homogeneous ensembles for study.

A V: *Pictorial representation of ensembles.* In quantum mechanics the probabilities associated with measurement results[**] are described either by a wave function Ψ_i or projector $|\Psi_i\rangle\langle\Psi_i| = \rho_i = \rho_i^2$, or by a density operator $(\rho > \rho^2)$ which is not a statistical mixture of projectors of the kind introduced by von Neumann[7], Jaynes[23], and Katz[24].

Pictorially, we can visualize a projector by an ensemble of identical systems, identically prepared. Each member of such an ensemble is characterized by the same projector ρ_i, and von Neumann calls the ensemble *homogeneous*. Similarly, we can visualize a density operator ρ consisting of a statistical mixture of two projectors $\rho = \alpha_1\rho_1 + \alpha_2\rho_2$, $\alpha_1 + \alpha_2 = 1$, $\rho_1 \neq \rho_2 \neq \rho$, where ρ_1 and ρ_2 represent quantum –mechanical probabilities, α_1 and α_2 statistical probabilities, and the ensemble is called *heterogeneous* or *ambiguous*[2]. A heterogeneous ensemble is shown in Figure A-1.

The concept of a mixture begs the questions: (i) Are there quantum-mechanical problems that involve probability distributions which cannot be described by a projector but require a purely quantum-mechanical density operator – a density operator which is not a mixture?; and (ii) Are there purely quantum-mechanical density operators consistent with the foundations of quantum physics?

Upon close scrutiny of the definitions, postulates, and key theorems of quantum theory, we find that the answers to both questions are yes[9,10]. A purely quantum-mechanical density operator $\rho > \rho^2$ is represented by an ensemble of identical systems, identically prepared, as shown in Figure A-2, and, by analogy to a projector, is called *homogeneous* or *unambiguous*[9]. Density operators that correspond to homogeneous ensembles have many interesting implications. They extend quantum ideas to thermodynamics, and thermodynamic principles to quantum phenomena.

For example, it is shown that thermodynamics applies to all systems (both large and small, including one particle systems, such as one spin), to all states (both thermodynamic equilibrium, and not thermodynamic equilibrium), and that entropy is not a measure of ignorance, lack of information or disorder[25] but an intrinsic property of a system[26] in the same sense that inertial mass is a property of a system. Again, it is shown that entropy is a measure of the quantum-mechanical spatial

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[*] We use the expression "probabilities associated with measurement results" rather than "state" because the definition of state (A XIV) requires more than the specification of a projector or a density operator.

shape of the constituents of a system, and that irreversibility is solely due to the changes of this shape as the constituents try to conform to the external and internal force fields of the system[11,12].

It is noteworthy that for given values of energy, volume, and amounts of constituents, if ρ is a projector the entropy $S = 0$, if ρ is not a projector but corresponds to a state that is not stable equilibrium (not thermodynamic equilibrium) S has a positive value smaller than the largest possible value, and if ρ corresponds to the unique stable equilibrium state S has the largest value. Said differently, all projectors or wave functions correspond to zero entropy physics, all largest entropy density operators for different isolated system specifications correspond to stable equilibrium states – classical thermodynamics – and all other density operators that are associated neither with zero entropy, nor with largest value entropy correspond to probability distributions that can be represented neither by projectors nor by stable equilibrium state density operators.

A VI: *Property*. The term *property* refers to any attribute of a system that can be quantitatively evaluated at an instant in time by means of measurements and specified procedures. All measurement results and procedures are assumed to be precise, that is, both error free, and unaffected by the measurement and the measurement procedures.

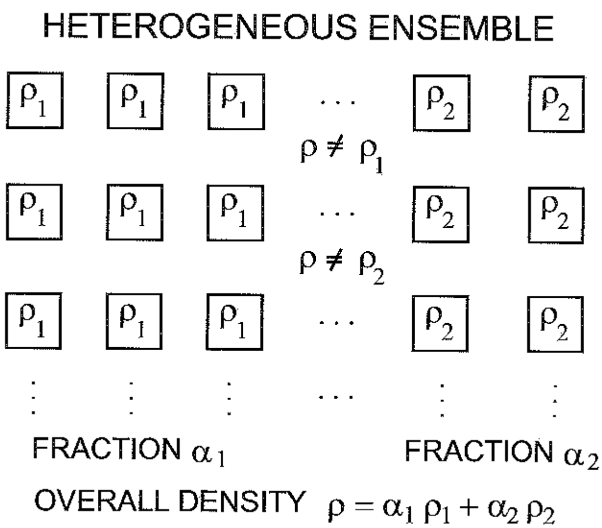

Figure A-1: Pictorial representation of a hetero-geneous ensemble. Each of the subensembles for ρ_1 and ρ_2 represents either a projector $\left(\rho_i = \rho_i^2\right)$ or a density operator $\left(\rho_i > \rho_i^2\right)$, for i = 1, 2, and $\alpha_1 + \alpha_2 = 1$.

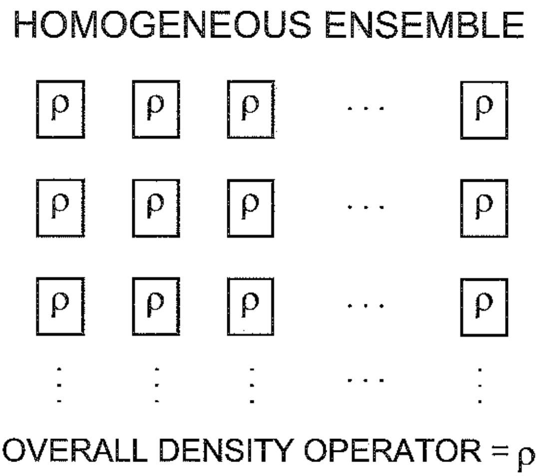

Figure A-2: Pictorial representation of a homogeneous ensemble. Each of the members of the ensemble is characterized by the same density operator $\rho \geq \rho^2$. It is clear that any conceivable subensemble of a homogeneous ensemble is characterized by the same ρ as the ensemble, and pertains to the same system at a given instant in time.

A VII: *Observable.* From A VI follows that each property can be observed. Traditionally, however, a property is called an *observable* only if it conforms to the following mathematical representations.

A VIII: *Correspondence postulate.* Some linear Hermitian operators A, B, ... on Hilbert space $\mathcal{H}$, which have complete orthonormal sets of eigenvectors, correspond to observables of a system.

The inclusion of the word "some" in the correspondence postulate is very important because it indicates that there exist Hermitian operators that do not represent observables, and properties that cannot be represented by Hermitian operators. The first category accounts for Wick et al[27] superselection rules, and the second accounts both for compatibility of simultaneous measurements introduced by Park and Margenau[20], and for properties, such as temperature, that are not represented by operators.

A IX: *Measurement act. A measurement act* is a reproducible scheme of measurements and operations on a member of an ensemble. Regardless of whether the measurement refers to an observable or not, in principle the result of such an act is presumed to be precise, that is, an error and perturbation free number.

If a measurement act of an observable is applied to each and every member of a homogeneous ensemble, the results conform to the following postulate and theorems.

A X: *Mean-value postulate.* If a measurement act of an observable represented by a Hermitian operator A is applied to each and every member of a homogeneous ensemble, there exists a linear functional m(A) of A such that the value of m(A) equals the arithmetic mean of the ensemble of A measurements, that is,

$$m(A) = \langle A \rangle = \sum_{i} a_i / N \quad \text{for } N \to \infty \qquad \text{(A-1)}$$

where a_i is the measurement result of a measurement act of A applied to the ith member of the ensemble, a large number (theoretically infinite) of a_i's have the same numerical value, and both m(A) and $\langle A \rangle$ represent the mean or expectation value of A.

A XI: *Mean-value theorem.* For each of the mean-value functionals or expectation values m(A) of a system at an instant in time, there exists the same Hermitian operator ρ such that

$$m(A) = \langle A \rangle = \mathrm{Tr}[\rho A] \qquad \text{(A-2)}$$

where Tr stands for the trace (sum of diagonal elements) of the operator that follows. The operator ρ is known as the *density operator* or the *density of measurement results of observables*, and here it can be represented solely by a homogeneous ensemble as shown in Figure A-2. It is noteworthy that the value $\langle A \rangle$ depends exclusively on the Hermitian operator A that represents the observable and on the density operator ρ, and not on any other operator that either commutes or does not commute with operator A.

The operator ρ is proven to be Hermitian, positive, unit trace and, in general, not a projector, that is,

$$\rho > 0; \qquad \mathrm{Tr}\rho = 1; \qquad \text{and } \rho \geq \rho^2 \qquad \text{(A-3)}$$

A XII: *Probability theorem.* If a measurement act of an observable represented by operator A is applied to each and every member of a homogeneous ensemble characterized by ρ, the probability or frequency of occurrence $W(a_n)$ that the results will yield eigenvalue a_n is given by the relation

$$W(a_n) = \mathrm{Tr}[\rho A_n] \qquad \text{(A-4)}$$

where A_n is the projection onto the subspace belonging to a_n,

$$A \,|\, a_n^{(i)} \rangle = a_n \,|\, a_n^{(i)} \rangle \qquad \text{(A-5)}$$

for n = 1, 2, ..., i = 1, 2, ..., g, and g is the degeneracy of a_n.

A XIII: *Measurement result theorem.* The only possible result of a measurement act of observable A is one of the eigenvalues of A (Eq. A-5).

A XIV: *State.* The term *state* means all that can be said about a system at an instant in time, that is, a set of Hermitian operators A, B, ... that correspond to $n^2 - 1$ linearly independent observables, and the values of these observables given by the relations

$$\langle A \rangle = \mathrm{Tr}[\rho A] = \sum_i a_i / N$$

$$\langle B \rangle = \mathrm{Tr}[\rho B] = \sum_i b_i / N \qquad \text{(A-6)}$$

$$\vdots \; \vdots \quad \vdots \quad \vdots \quad \vdots$$

where n is the dimensionality of the Hilbert space, and $N \to \infty$.

In Eqs. (A-6), either the density operator ρ is specified *a priori* and the values of the observables are calculated, or the values $\sum_i a_i/N$, $\sum_i b_i/N$, ... of the linearly independent observables are either specified or experimentally established and a unique operator ρ is calculated. The mappings both from ρ to expectation values and from expectation values to ρ are unique because Eqs. (A-6) are linear from expectation values to ρ and from ρ to expectation values. It is noteworthy that only the first power of an operator X and its eigenvalues x_i are included in Eqs. (A-6). For example, only the Hamiltonian operator H and its eigenvalues ϵ_1, ϵ_2, ..., appear in Eqs. (A-6), and not H^m and its eigenvalues ϵ_1^m, ϵ_2^m, ... for $m > 1$.

Finally, it is clear that the definition of state is not synonymous either with the concept of a wave function or more generally the concept of a density operator (A XI). The definition of state requires both the specification of the system (A I), and either a complete set of measurable, independent expectation values, or a prescribed density operator $\rho \geq \rho^2$.

A XV: *Uncertainty relations.* Ever since the inception of quantum mechanics, the uncertainty relation that corresponds to a pair of observables represented by non-commuting operators is interpreted by many scientists and engineers as a limitation on the accuracy with which the observables can be measured[28-30]. This interpretation, however, cannot be deduced from the postulates and theorems of quantum theory. An outstanding example of measurement accuracy is the Lamb shift[31].

The probability theorem avers that we cannot predict which precise eigenvalue each measurement will yield except in terms of either a prespecified or a measurable probability or frequency of occurrence. Each probability distribution of an observable represented by operator X has a variance $(\Delta X)^2$ and a standard deviation ΔX. Moreover, for two observables represented by two non-commuting operators A and B, that is, $AB - BA = iC$, it is shown[10,32] that ΔA and ΔB satisfy the uncertainty relation $\Delta A \Delta B \geq |\langle C \rangle / 2|$, that is, each uncertainty relation refers neither to any errors introduced by the measuring instruments nor to any particular value of a measurement result.

A XVI: *Collapse of the wave function postulate.* Among the postulates of quantum mechanics, many authoritative textbooks include von Neumann's projection or collapse of the wave function postulate[30,32,33] which is proven not to be valid[16].

DYNAMICS: EVOLUTION OF THE DENSITY OPERATOR IN TIME

A XVII: *Dynamical postulate.* Hatsopoulos and Gyftopoulos[9] postulate that unitary evolutions of ρ in time obey the relation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H\rho - \rho H\right] \qquad \text{(A-7)}$$

Though Eq. (A-7) is well known in the literature as the von Neumann equation, here it must be postulated for the following reason. In statistical quantum mechanics[34] the equation is derived as a statistical average of Schrödinger equations, each of which describes the evolution in time of a projector ρ_i and each of which is multiplied by a time independent statistical probability α_i. But here, ρ is not a mixture of projectors and therefore cannot be derived as a statistical average of projectors. It is noteworthy that the dynamical postulate (Eq. A-7) is incomplete because it describes reversible adiabatic processes, but not all reversible adiabatic processes are unitary[9], and not all processes are reversible.

A nonlinear equation of motion of ρ that describes both all reversible processes and all irreversible processes has been conceived by Beretta et al[21,22]. It is not discussed here for the sake of brevity. The only thing we wish to emphasize is that the Beretta equation is shown to satisfy all the requirements for it to be a bona fide equation of motion of a nonstatistical unified theory of quantum mechanics and thermodynamics[35,36,37,38].

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