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ON THE MEASUREMENT OF DYNAMIC CHARACTERISTICS OF NUCLEAR REACTOR SYSTEMS*

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Summary

The problem of understanding the dynamics of a nonlinear physical system, from an engineering standpoint, is often approached by means of the powerful technique of input-output measurements. For example, the dynamics of a nuclear reactor system are explored by varying the position of a control rod in a predetermined or statistically controlled manner and measuring the corresponding variations of the power output.

There are two questions that are related to the interpretation and use of input-output data when one is concerned only with the problem of analysis. The first question is: given a set of input-output measurements, how can one extract from them enough information to establish a mathematical model for the system? The second question is: given a physical system, like a reactor, what is the best input to use so that one set of input-output measurements contains all the information about the system?

To answer these two questions, consider a physical, stationary system. The output $y(t)$ is a functional of the past history of the input $x(t)$:

$$y(t) = F\big(x(t-\tau)\big) \qquad \tau < t \tag{1}$$

This functional can be expanded into a sum of functionals. In particular, if $\phi_n(t)$ is an arbitrary orthonormal complete set, then:

$$x(t-\tau) = \sum_i u_i(t)\phi_i(\tau) \qquad u_i(t) = \int_0^\infty x(t-\tau)\phi_i(\tau)d\tau \tag{2}$$

and consequently:

$$y(t) = F(u_1(t), u_2(t)\ldots u_n(t)\ldots) =$$

$$= a_0 + \sum_i a_i u_i(t) + \sum_{i,j} a_{ij} u_i(t) u_j(t) + \ldots +$$

$$+ \sum_{i,j..k} a_{ij..k} u_i(t) \ldots u_k(t) + \ldots =$$

$$= h_0 + \int_0^\infty h_1(\tau)x(t-\tau)d\tau + \int_0^\infty\int_0^\infty h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \tag{3}$$

$$+\ldots+\int_0^\infty ..\int_0^\infty h_k(\tau_1,\tau_2..\tau_k)x(t-\tau_1)..x(t-\tau_k)d\tau_1..d\tau_k + \ldots$$

where $h_k(\tau_1,\tau_2..\tau_k) = \sum_{i,j..k} a_{ij..k}\phi_i(\tau_1)\ldots\phi_k(\tau_k)$. The coefficients $a_{ij..k}$ or the kernels $h_k(\tau_1,\ldots\tau_k)$ differ from system to system. Expansion (3) is possible because the coefficients $u_i(t)$, an equivalent transform of $x(t-\tau)$, are functions of the present time only.

The important implication of Eq. (3) is that the output of a nonlinear physical system can be described by a sum of generalized convolution integrals or, in other words, as a sum of functionals of the input. This suggests that if the functionals were orthogonal it would be easy to extract the characteristic kernels $h_k(\tau_1,\ldots\tau_k)$ from a single set of input-output data.

Indeed, consider the sequence of all the past values of a particular input $x(t)$. Assume that it is possible to use this sequence to form a complete set of homogeneous orthogonal polynomials $P_n(x(t,\tau))$, $n = 0,1,2\ldots$ (1). Expand the output in terms of functionals of these polynomials:

$$y(t) = H_0 + \int_0^\infty H_1(\tau)P_1(x(t,\tau))d\tau + \int_0^\infty\int_0^\infty H_2(\tau_1,\tau_2)P_2(x(t,\tau_1,\tau_2))d\tau_1 d\tau_2 +$$

$$+ \ldots + \int_0^\infty \ldots \int_0^\infty H_k(\tau_1 \ldots \tau_k)P_k(x(t,\tau_1 \ldots \tau_k))d\tau_1 \ldots d\tau_k + \ldots \qquad (4)$$

It is easily shown that the kernels $H_n(\tau_1,\ldots\tau_n)$ can be assumed symmetrical without any loss of generality and that the functionals in the right hand side of Eq. (4) are orthogonal, regardless of the value of the kernels $H_n(\tau_1,\ldots\tau_n)$. The consequence of the orthogonal expansion (4) is that the kernels $H_k(\tau_1,\tau_2\ldots\tau_k)$ can be measured experimentally by the arrangement shown in Fig. 1. If the "known box" has an output which is:

$$y_o(t) = \int_0^\infty \ldots \int_0^\infty \delta(\tau_{n+1}-\tau_1)\ldots\delta(\tau_{2n}-\tau_n)P_n(x(t,\tau_1,\ldots\tau_n))d\tau_1\ldots d\tau_n \qquad (5)$$

the output of the integrator is:

$$I = H_n(\tau_n,\tau_{n+1}\ldots\tau_{2n}) \qquad (6)$$

It has been shown (2) that the most general input for a nonlinear system is a gaussianly distributed, white noise signal. The corresponding orthogonal polynomials are the Hermite polynomials (3) and the "known boxes" can be built with delay lines and multipliers.

However, the simplicity of the gaussian, white noise signal is of no practical avail because of the extremely long integration time required to avoid errors. Other broad band but not necessarily gaussian signals can be used to measure the different kernels. Of course, then the "known boxes" may be more involved.

An immediate consequence of the orthogonal expansion (4) is that oscillation and autocorrelation measurements in nuclear systems yield results different from each other and different from the linear transfer function of the system. On the other hand the crosscorrelation method (4) measures only the linear transfer function of the expansion. These points will be illustrated during the presentation with experimental results.

References

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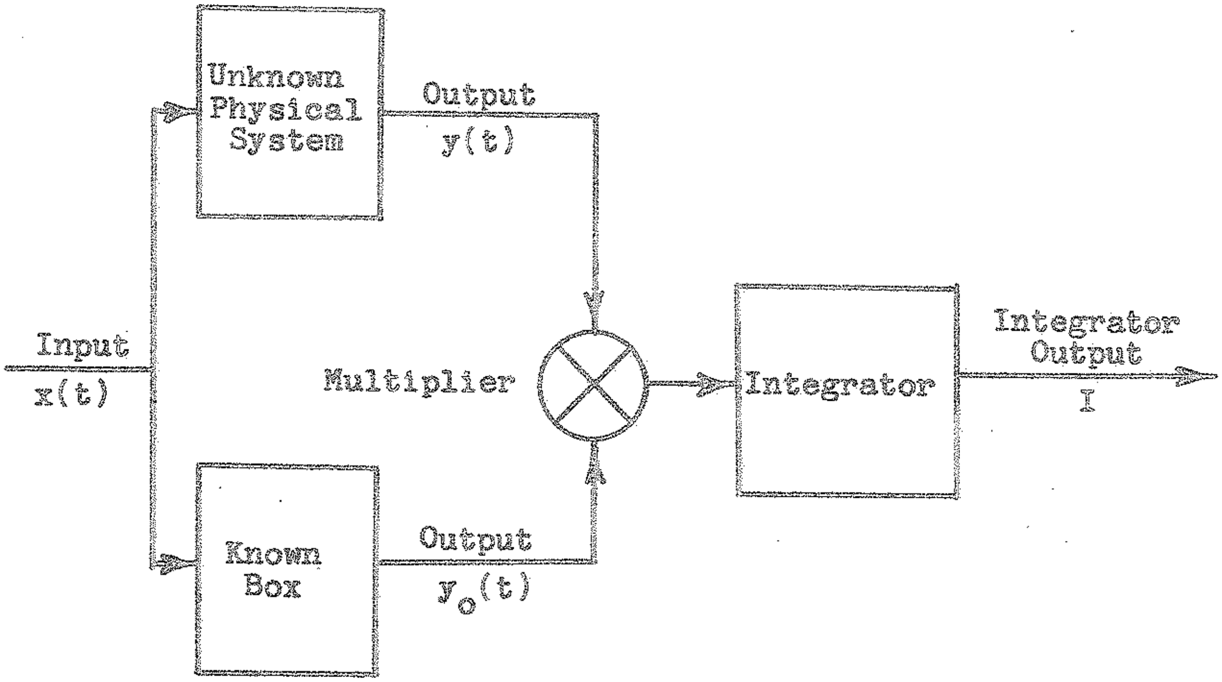

Figure 1. Experimental arrangement for the measurement of the characteristic kernels of a nonlinear physical system.