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SIGNALS FOR TRANSFER FUNCTION MEASUREMENTS IN NON-LINEAR SYSTEMS

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The purpose of this paper is to describe some work on the synthesis of test signals for the measurement of kernels in non-linear systems.

Given an input $x(t)$ of a stationary lumped parameter non-linear system, the output $y(t)$ can be represented as a functional of the past history of the input $y(t) = F[x(t-\tau);\ \tau < t]$. This functional can always be expanded into a sum of functionals (Wiener, 1958; Gyftopoulos, 1963):

$$y(t) = h_0 + \int_0^\infty h_1(\tau)x(t-\tau)\,\mathrm{d}\tau + \int_0^\infty\int_0^\infty h_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 + \int_0^\infty\int_0^\infty\int_0^\infty h_3(\tau_1, \tau_2, \tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\mathrm{d}\tau_3 + \cdots \tag{1}$$

where h_i are kernels characteristic of the system in question and, without loss of generality, may be assumed to be symmetrical.

In many practical applications it is of particular interest to know the linear kernel (h_1) whose spectrum is the transfer function of the first approximation of the system. In view of the noise present in physical systems, the question arises as to whether an input can be designed that permits the measurement of h_1 without restriction on the amplitude of $x(t)$.

To this end a number of periodic signals have been designed that allow the exact measurement of h_1, by means of cross-correlation measurements between input and output, in non-linear systems which can be satisfactorily described by a functional representation of the form

$$y(t) = h_0 + \int_0^\infty h_1(\tau)x(t-\tau)\,\mathrm{d}\tau + \int_0^\infty\int_0^\infty h_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 \tag{2}$$

One family of periodic signals is defined by the following properties:

(a) At any instant of time the signal may assume only one of three possible values: $+1$, 0, or -1.
(b) The signal is discontinuous and may change value at event points having uniform spacing Δt.
(c) The signal is periodic with period $T = N\,\Delta t$, where $N = 3^n - 1$, n = integer.
(d) The j-th bit is given by the recursion formula

$$C_j = a_1 C_{j-1} + a_2 C_{j-2} + \cdots + a_n C_{j-n};\ a_i = \text{integer} \tag{3}$$

where the right side is reduced modulo-3 so that C_j assumes the values $+1$, 0, or -1. Given n, signals with period $N\,\Delta t$ exist only for certain values of the coefficients $a_1, a_2, \ldots, a_n$.

Similar families of signals can be designed with $N = p^n - 1$, where p is a prime integer greater than 2 and equal to the number of possible levels of the signal.

(e) If the n-th order correlation function is defined as

$$\phi_n(\tau_1, \ldots, \tau_n) = \int_{\tau/2}^{\tau/2} x(t)x(t + \tau_n)\,dt \tag{4}$$

then the preceding families of signals have a first-order correlation function as shown in Fig. 1, and a second-order correlation function which is zero everywhere.

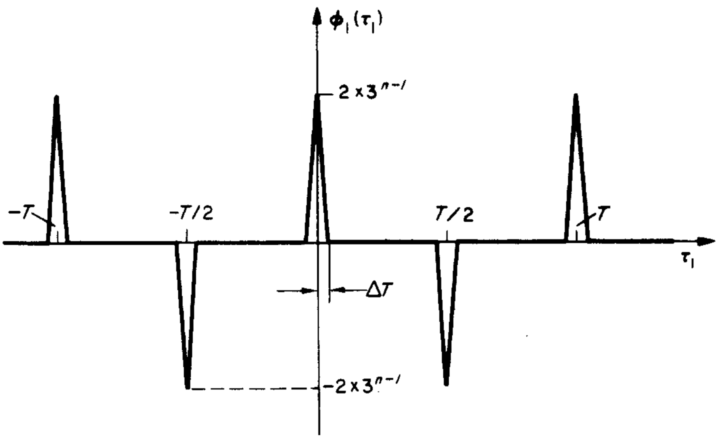

FIG. 1.—First-order correlation function of periodic ternary sequence.

For signals of this family it is easily established that the cross-correlation of $y(t)$ of equation (2) with $x(t)$, over an integer number of periods, yields

$$\int_0^{kT} x(t - \tau_1)y(t)\,dt \propto h_1(\tau_1); \quad k = \text{integer} \tag{5}$$

provided that h_1 is negligibly small for $\tau_1 > T/2$ (BALCOMB *et al.*, 1961).

The family of signals discussed above can be used for reactor transfer function measurements with better accuracy than is attainable with the binary signals presented in BALCOMB *et al.* (1961).

Efforts are currently under way to design signals whose higher order correlation functions closely approximate the corresponding correlation functions of Gaussian white noise (WIENER, 1958). The objectives of these efforts are the measurement of h_1 in the presence of kernels of order three and higher as well as the measurement of the higher order kernels themselves.

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