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REACTOR TRANSFER FUNCTION DETERMINATION AND APPLICATION

Chairman: Perhaps instead of taking a break now, it might be best to pass on to a few of the panelists, and then take a break a little later, when the ball game gets interesting. Our first panelist will be Mr. Gyftopoulos from MIT.

Professor Gyftopoulos: I would like to talk about a statistical stability monitor. The title I am using is for "commercial purposes" because I know that Dr. Kouts is going to describe something similar.

The monitor is based on the method of using externally applied statistical signals to get information about the transfer function of a system. Currently an MIT graduate student is working on the method at Los Alamos, and I think today, if things run on schedule, he will be trying it on one of the Rover reactors.

Basically the method consists of the following: Suppose that we have a system which is characterized by a system function h(t) and that we wish to determine this function experimentally. Then, as we heard from the previous speaker, there are several methods, such as oscillation tests or correlation of inherent statistical fluctuations, which we can use to derive the desired information, either in the frequency or in the time domain.

Another method, relatively new in the nuclear reactor field, is to excite the system under investigation by an externally applied random input R(t), measure the corresponding output O(t) and crosscorrelate the input with the output. It turns out--and I will not go into the details--that the crosscorrelation of the input and output is equal to the convolution integral of the system function and the autocorrelation function of the input:

$$\phi_{RO}(\tau) = \int_0^T h(t)\phi_{RR}(\tau-t)dt \qquad (1)$$

where $\phi_{RO}(\tau)$ = the crosscorrelation of input and output over a period T

$\phi_{RR}(\tau)$ = the autocorrelation of the input over a period T

T = the settling time of the system function.

If the random input is chosen appropriately, and in particular if it is chosen so that the autocorrelation function has an impulse-like shape (delta function), then the convolution integral reduces effectively to the value of the system function at time τ:

$$\phi_{RO}(\tau) = h(\tau) \qquad (2)$$

In other words the crosscorrelation function is equal to the system function.

A random function with an impulse-like autocorrelation function can be created by several methods. One way is to use a binary chain which resumes values plus or minus one unit with equal probability and may (but need not necessarily) change sign at event points spaced at Δt seconds apart. If the correlation time were infinite, this binary chain would have a triangular autocorrelation function of half-width Δt. However, since in any practical case the correlation time is finite, in general the autocorrelation function is more or less as shown in Figure 1. In order to avoid the side lobes of the autocorrelation function, which are sources of unpredictable errors, one can use special idealized binary chains. Such chains have been developed and, among other requirements, they fulfill the condition that they are periodic of period $T = N\Delta t$ where $N = 251$, 1019, etc. The autocorrelation function of an idealized chain is shown in Figure 2. The constant value of the autocorrelation beyond the spike is relatively small and can be easily accounted for in the experimental results. In particular:

$$\phi_{RO}(\tau) = h(\tau) - \frac{1}{T}\int_0^T h(t)dt \qquad (3)$$

The meaning of Eqs. (1) or (3) can be graphically interpreted as shown in Figure 3. Suppose that the system function is somewhat as in Figure 3a. If the autocorrelation function of the input is as in Figure 3b, the convolution integral (1) implies effectively that the system function is sampled at the time τ, determined by the delay time of the cross- or autocorrelation. Notice that the correction term in Eq. (3) can be determined by using a delay time $\tau - \Delta t$.

Well, this is the essence of the crosscorrelation method for the derivation of the system function. The latter, of course, can be easily transformed into the frequency domain to yield the transfer function. Now it is a simple matter to use this method and build a stability monitor. More specifically, assume that a system is excited by an idealized binary chain as shown in Figure 4. The input is delayed, say by an amount τ_j, and multiplied by the output. The product is fed into an integrator which integrates over a period T. The output of the integrator is approximately the value of the system function $h(\tau_j)$ at the time τ_j. By introducing a series of delays, of different values, and a series of multipliers and integrators in parallel, we can have a series of points of the system function. These points can be displayed on a screen to provide a continuous visual record of the system function. From the behavior of the system function we can easily infer whether the system is approaching instability or not as it was pointed out by Dr. Thie.

Equipment of the type suggested by Figure 4 has been built by my student, Mr. Balcomb, at Los Alamos. This equipment has been tested on Godiva followed by a second order filter and the results were very satisfactory. The equipment was also tried on several simulated reactor systems with equally satisfactory results. Today or in a couple of days the same equipment is scheduled for use to study the dynamic characteristics of KIWI-A3 at the Nevada test site. I hope to report to you the experimental results in the near future.

The crosscorrelation method has several advantages over other methods used for the measurement of the system function of nuclear reactor systems.

First, one can get the entire information about the system function h(t) in a very short time, as opposed to the case of oscillation tests. This is due to the fact that when the crosscorrelation method is used all the frequencies of the spectral density of the reactor system are excited at once while in the case of oscillation tests each frequency must be excited separately.

Second, the crosscorrelation method is not as limited as other methods by the presence of internal noise. The reason why this is true is because the random input has different statistical properties than the inherent noise and therefore during the crosscorrelation process the noise part of the output, due to internal noise, crosscorrelates out. This property is somewhat limited by the finite time of correlation; however, the experimental results so far show that the limitation is not very serious.

Third, since the crosscorrelation method is not limited by internal noise, one can introduce a very small amplitude signal and perform the experiment safely and without consideration of nonlinearities. Of course the smallness of the input is a function of the correlation time (if you like, it is a function of the error that can be tolerated). The indications are that in a really noisy system the input may have to be as large as the inherent noise. However, this can be traded with correlation time.

A fourth minor advantage is that the experimental equipment necessary for the use of the crosscorrelation method, whether used for transfer function measurement or as a stability monitor, can be accomodated in a small package, say, 10" x 10" x 20". Los Alamos is planning to build such a device and have it used for reactors which are available for testing only for a short period of time.

Now, there was another comment that I wanted to make, which I forgot--no, I guess that's all.

<u>Question from Floor</u>: Isn't this the same system essentially that Goodwin developed for testing a cat-cracker several years ago?

<u>Prof. Gyftopoulos</u>: I am not aware of this particular development, but I wouldn't be surprised because this development came out of some work that Professor Lee did at MIT for the statistical study of communication systems. He made use of correlation techniques for cases where the signals involved are characterized by statistical properties

rather than analytical functions. The method is also well known in the field of automatic control.

The comment that I wanted to make earlier is that on account of mechanical limitations of the equipment and economic considerations, it may not be easy to evaluate a system by this method if the bandwidth of the system is very large. Now, by "large" here, one means about 50 cycles per second. However, in case you are interested in the impulse response of a system as a whole, like a nuclear reactor system, the bandwidth is usually not that large. It may be of the order of 10 cycles per second, and then the method is applicable with no need for elaborate electronic instrumentation.

Question: It has one disadvantage for a power reactor application, which is that you have to have a disturbance mechanism operating in the reactor continually, and this may not be too convenient in large power stations.

Prof. Gyftopoulos: Yes, but if--and I'm not certain yet how much reactivity it would require--if the noise doesn't bother you and if the signal you have to put in is not very large, it seems to me that it wouldn't be very difficult to put a small control rod in the reactor, which you oscillate.

Comment from Floor: Or you could oscillate the flow or something else.

Prof. Gyftopoulos: You could do it this way and you introduce the kind of versatility that Dr. Thie was talking about. Of course, you would get a different kind of transfer function--but this serves the purpose just as well.

Comment from Floor: If it's noise in the system, it doesn't matter much in which variable you put it on.

Prof. Gyftopoulos: Exactly, but you would have a different kind of transfer function.

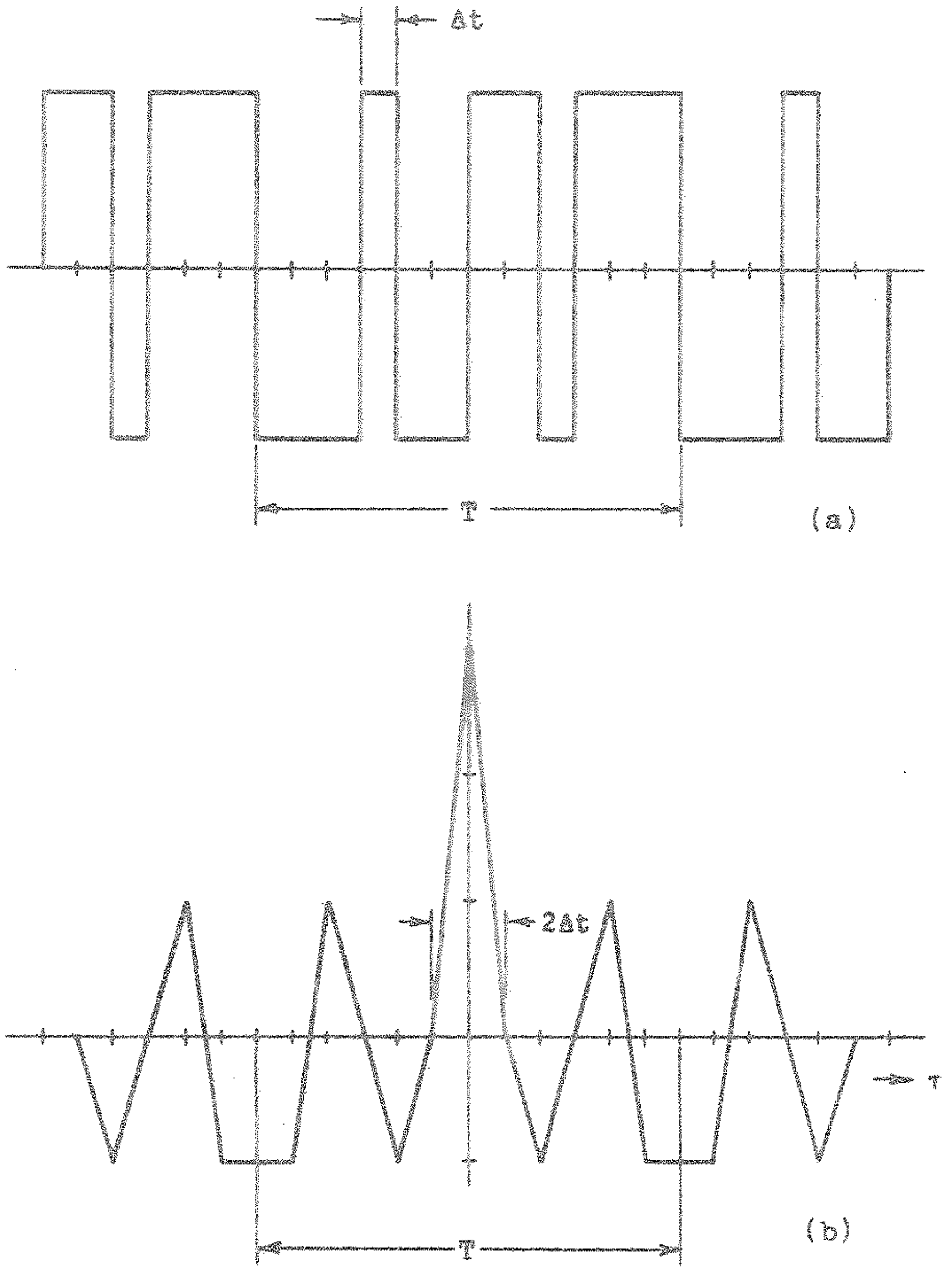

Figure 1. Arbitrary random binary signal (a) periodic over T and autocorrelation function (b) of the same. The autocorrelation side lobes are introduced by the finite correlation time T.

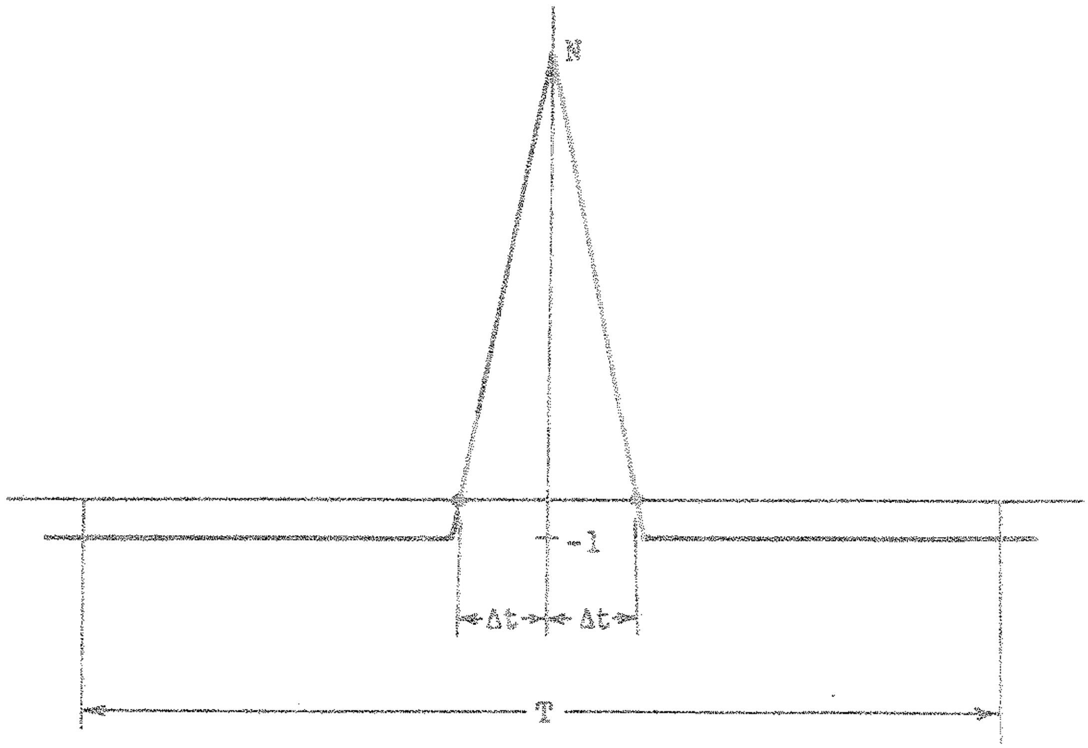

Figure 2. Autocorrelation function of an "idealized" random binary chain, periodic over T ($T = N\Delta t$, $N = 251$ or 1019 etc).

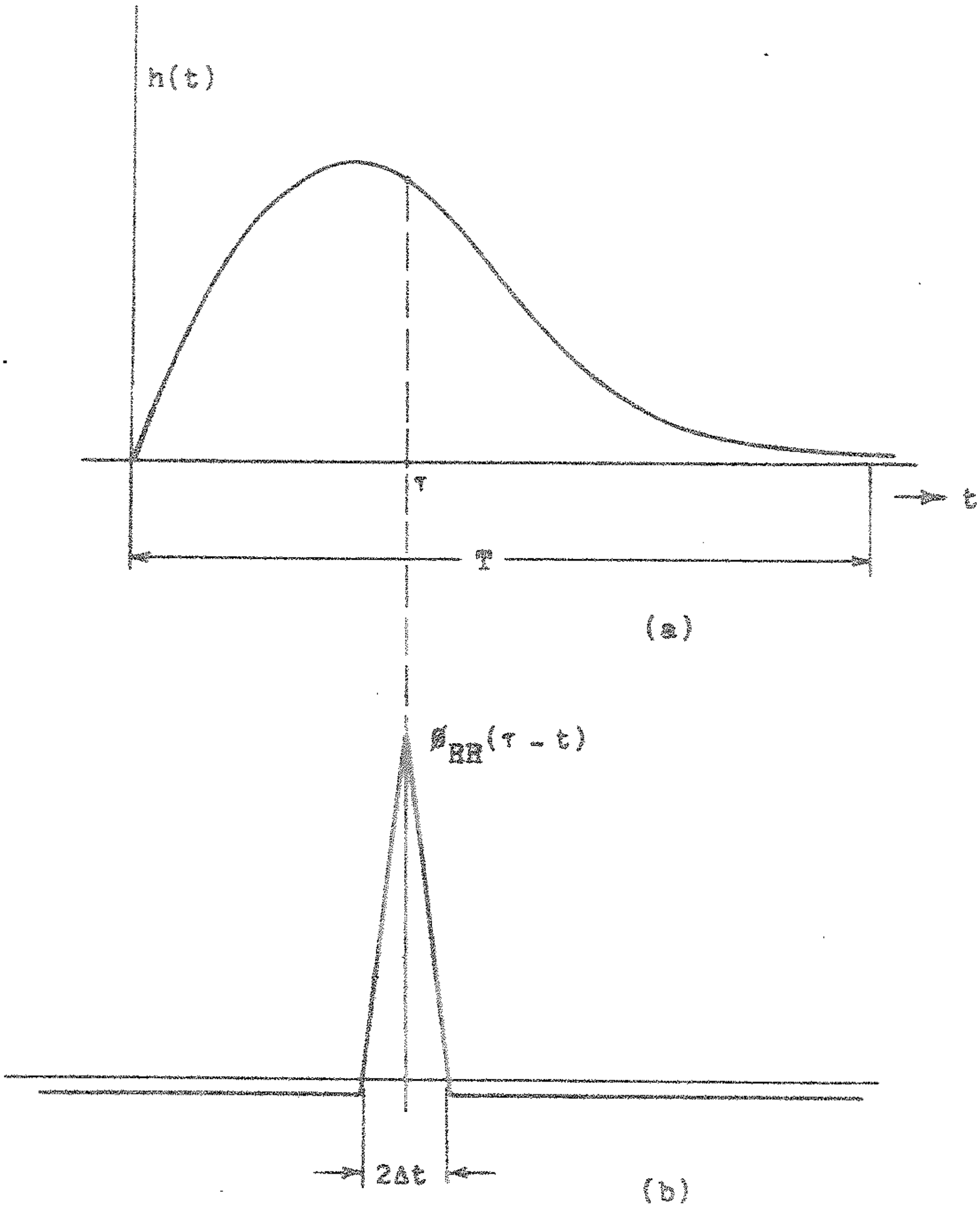

Figure 3. Typical system function h(t) (a) and autocorrelation $\phi_{RR}(\tau - t)$ of an "idealized" random binary chain (b). The convolution of h(t) and $\phi_{RR}(t)$ is equivalent to sampling h(t) by the window function $\phi_{RR}(\tau - t)$. This sampling yields $\sim h(\tau)$.

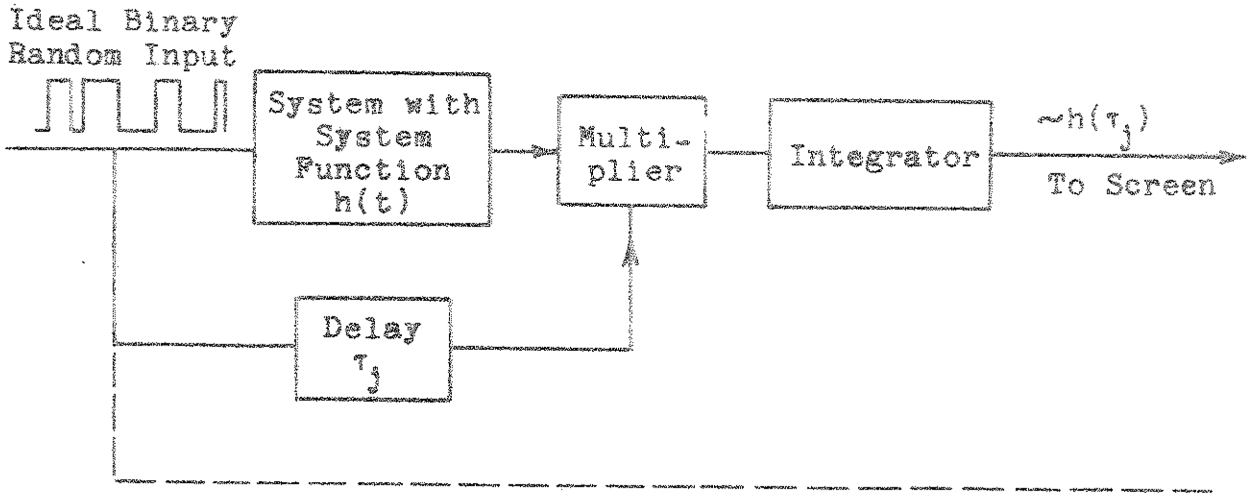

Figure 4. Schematic of stability monitor or of experimental setup for the measurement of the system function by the crosscorrelation method.