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POINT REACTOR KINETICS AND STABILITY CRITERIA

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1. Introduction

During the past few years efforts have been devoted to the understanding of the capability and limitations of point reactor kinetics with regard to their relationship to experiments. [1 - 3] Even though it is generally recognized that point kinetics fail to account for the dynamic behavior of large core reactors, it is not equally well accepted that similar failures may arise in small reactors where relatively small flux shape changes may occur over short periods of time.

One of the purposes of this paper is to re-derive the kinetics equations in an effort to illustrate these failures and to emphasize the need for space-time dependent kinetics for meaningful correlations of experimental measurements. A positive result of this re-derivation is the recognition that point kinetics can always be used to answer questions of overall stability or instability. Thus, the second purpose of the paper is to review existing criteria for nonlinear stability, to indicate how restrictive they are and then suggest some practical considerations which might bridge the discrepancy between analysis and experience.

2. Point Reactor Kinetics

2.1 Transport Theory and Conventional Neutron Kinetics

The most important variable in reactor dynamics is the fission energy stored in the reactor and its distribution because this energy can be directly related to all other variables and all material and environmental constraints. Even though the stored

energy is such a key variable, it has not been possible to measure it and control it directly. Instead, the energy behavior is inferred from measurements of the neutron population, the temperatures of different components, the pressure etc.

The analysis of the time behavior of the neutron population in a reactor can be treated by a number of alternative models. A basic model is the one implied by transport theory. This theory allows a precise and detailed representation of all conceivable interactions that neutrons undergo in a reactor with a minimum number of assumptions. The equations that result, however, are very difficult to handle analytically. Consequently, they need to be reduced to simpler forms and then be related in an approximate manner to the energy stored in the reactor. To this end, consider a reactor with a stationary fuel. For mathematical simplicity and without any loss of generality, assume that there is only one species of fissionable material and that there is a very large number of neutrons so that statistical fluctuations can be ignored.

The time dependent balance equations for the neutron population and the delayed neutron precursors are:

$$\frac{\partial}{\partial t}N(\bar{r},E,\bar{\Omega},t) =$$

$$\frac{1}{4\pi}f_o(E)\int_{\bar{\Omega}'}d\bar{\Omega}'\int_o^{\infty}dE'\,\nu(E')(1-\beta)v'\Sigma_f(\bar{r},E',t)N(\bar{r},E'\bar{\Omega}',t) +$$

$$+\int_{\bar{\Omega}'}d\bar{\Omega}'\int_o^{\infty}dE'v'\Sigma_s(\bar{r},E'\rightarrow E,\bar{\Omega}'\longrightarrow\bar{\Omega},t)N(\bar{r},E',\bar{\Omega}',t)$$

$$+\frac{1}{4\pi}\sum_{i=1}^{m}\lambda_i f_i(E)C_i(\bar{r},t) + S(\bar{r},E,\bar{\Omega},t) -$$

$$+\ \bar{\Omega}\,v\cdot\mathrm{grad}N(\bar{r},E,\bar{\Omega},t) - v\Sigma_t(\bar{r},E,t)N(\bar{r},E,\bar{\Omega},t)\ ; \qquad (1)$$

$$\frac{\partial}{\partial t}C_i(\bar{r},t) = \int_{\bar{\Omega}'}d\bar{\Omega}'\int_o^{\infty}dE'\,\nu(E')\beta_i v'\Sigma_f(\bar{r},E',t)N(\bar{r},E',\bar{\Omega}',t) -$$

$$-\ \lambda_i C_i(\bar{r},t)\quad ;\quad i = 1,2,\ldots m\ , \qquad (2)$$

where $N(\bar{r},E,\bar{\Omega},t)$ is the neutron density per unit volume, per unit relative neutron energy, E, per unit solid angle, $\bar{\Omega}$, at time t

and $C_i(\bar{r},t)$ is the concentration of the ith delayed neutron precursor per unit volume at position $\bar{r}$ and time t. The other symbols have their common usage as described in [1].

The difficulties of Eqs. (1-2) stem both from their dependence on seven independent variables, $(\bar{r},E,\bar{\Omega},t)$, and from the fact that the macroscopic cross sections are complicated implicit functions of the behavior of the neutron population throughout the entire reactor. Customarily an attempt is made to reduce Eqs. (1-2) to a set of ordinary differential equations. The reduction is achieved by a variety of mathematical techniques. Which technique is more appropriate depends on such considerations as computational expediency, reactor type and/or the particular aspect of dynamic behavior which is under investigation. The following technique has received particular attention. The neutron density is expressed in the form

$$N(\bar{r},E,\Omega,t) = G(t)N_g(\bar{r},E,\bar{\Omega},t) , \qquad (3)$$

where $G(t)$ is a time dependent function accounting for all growth or decay tendencies of the neutron density while $N_g(\bar{r},E,\bar{\Omega},t)$ is a shape function which is bounded at all times and for all values of $(\bar{r},E,\bar{\Omega})$ and accounts for the distribution of neutrons in the phase space $(\bar{r},E,\bar{\Omega})$. With this definition for $N(\bar{r},E,\bar{\Omega},t)$ and through consideration of the adjoint density, $N_o^*(\bar{r},E,\bar{\Omega})$, of an arbitrary source-free critical reference reactor, Eqs. (1-2) are reduced to the form:

$$\frac{dG(t)}{dt} = \left(\frac{\rho}{\Lambda} - \frac{\beta_e}{\Lambda}\right)G(t) + \sum_i^m \lambda_i C_i(t) + Q(t) - \left[\frac{\partial}{\partial t}\ln\int_V d\bar{r}\int_o^\infty dE\int_{\bar{\Omega}} d\bar{\Omega}\, N_o^*(\bar{r},E,\bar{\Omega})N_g(\bar{r},E,\bar{\Omega},t)\right] G(t) ; \qquad (4)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{ie}}{\Lambda}G(t) - \lambda_i C_i(t) + \left[\frac{\partial}{\partial t}\ln\int_V dr\int_o^\infty dE\int_{\bar{\Omega}} d\bar{\Omega}\, N_o^*(\bar{r},E,\bar{\Omega})N_g(\bar{r},E,\bar{\Omega},t)\right] C_i(t) , \qquad (5)$$

where the analytical expressions for $C_i(t)$, $Q(t)$, ρ/Λ and β_{ie}/Λ are given in [1].

The selection of the bounded shape function is arbitrary. One possibility is to normalize $N_g(\bar{r},E,\bar{\Omega},t)$ so that

$$\int_V d\bar{r}\int_0^\infty dE\int_{\bar{\Omega}} d\bar{\Omega}\, N_o^*(\bar{r},E,\bar{\Omega})N_g(\bar{r},E,\bar{\Omega},t) = \text{constant.} \quad (6)$$

Thus, the logarithmic term disappears from Eqs. (4-5) and G(t) describes the time dependent amplitude of the component of $N(\bar{r},E,\bar{\Omega},t)$ which has the same shape as the distribution of neutrons in the critical reference reactor. This normalization is practical when the neutron shape distribution does not change appreciably and/or rapidly because the solutions of Eqs. (4-5) can then be correlated unambiguously with the recording of any counter placed around or inside the reactor. Under these conditions G(t) can be interpreted as the average power of the reactor. The correlation is quite good because the important coefficient ρ/Λ in Eq. (4) is a bilinear average of the differences of cross sections between those of the reactor under consideration and those of the critical reference reactor. As such it can be estimated to a good degree of accuracy. On the other hand, if the neutron shape distribution changes appreciably or very rapidly, assuming that Eq. (6) is valid renders the correlation of G(t) with the reading of a counter ambiguous. The reason is that G(t) is proportional to the amplitude of the fundamental component of the flux while the counter reading depends on the behavior of the flux at the position of the counter. If an attempt is made to include the logarithmic term in Eqs. (4-5), then again the correlation becomes difficult both because knowledge of the temporal behavior of the shape function is needed and because association of any counter reading with an average flux requires a time dependent calibration of the counter.

Another technique which may be used to reduce Eqs. (1-2) into a set of ordinary differential equations is to write

$$N(\bar{r},E,\bar{\Omega},t) = P(t)N_p(\bar{r},E,\bar{\Omega},t)\ , \quad (7)$$

multiply Eqs. (1-2) by the fission cross-section and integrate over all $(\bar{r},E,\bar{\Omega})$. Thus, with appropriate definitions of ρ/Λ, β_{ie}/Λ, $C_i(t)$ and Q(t) a set of equations similar to Eqs. (4-5) is found in terms of P(t) instead of G(t). If the shape function

is chosen so that

$$\int_V d\bar{r} \int_0^\infty dE \int_{\bar{\Omega}} d\bar{\Omega}\, \Sigma_f(\bar{r},E,t) N_p(\bar{r},E,\bar{\Omega},t) = \text{constant}, \qquad (8)$$

the logarithmic term is missing from the equations and P(t) is a measure of the average power of the reactor. The difficulty with this formulation is that during appreciable or rapid changes of the neutron density shape function, $N_p(\bar{r},E,\bar{\Omega},t)$, the average power P(t) cannot be correlated with the reading of a counter. In addition, the coefficient ρ/Λ for this case is a bilinear average of the absolute cross-sections and consequently subject to large errors.

Finally, a third alternative is to derive a set of ordinary equations by using the counter cross-section as a weighting function instead of $N_o^*(\bar{r},E,\bar{\Omega})$ or $\Sigma_f(\bar{r},E,t)$. Thus a set of equations is established which describes directly the counter reading but again, during appreciable or rapid changes of the shape of the neutron density, the counter reading cannot be interpreted as measuring a physically meaningful quantity.

In summary, the essence of all these arguments is that the standard form of point kinetics may not be suitable for rapid transients.

2.2 Illustrative Example

Drs. A.F. Henry and S. Kaplan and Mr. J. Yasinsky of Westinghouse performed and contributed the following digital computer experiment for the purposes of this paper. They considered two slab reactors, 60 cm. and 240 cm. long, respectively, with material properties as shown in Table I. They computed the space-time dependent fluxes, $\phi_1(x,t)$ and $\phi_2(x,t)$, on the basis of a two group, diffusion theory approximation, using the data also shown in Table I. Delayed neutrons were neglected since the time of interest was only 10 ms. The computed fluxes were used to find the following quantities:

$$\psi(x,t) = \frac{\langle \psi^*(x,0), \frac{1}{v_2}\phi_2(x,0)\rangle}{\langle \psi^*(x,0), \frac{1}{v_2}\phi_2(x,t)\rangle}\, \phi_2(x,t), \qquad (9)$$

$$G(t) = \frac{<\psi^*(x,0), \frac{1}{v_2}\phi_2(x,t)>}{<\psi^*(x,0), \frac{1}{v_2}\phi_2(x,0)>}, \qquad (10)$$

$$P(t) = \frac{<\nu\Sigma_f, \psi(x,t)>}{<\nu\Sigma_f, \psi(x,0)>}, \qquad (11)$$

$$C(t) = \frac{<\Sigma_c, \psi(x,t)>}{<\Sigma_c, \psi(x,0)>} = \frac{\psi(z,t)}{\psi(z,0)}, \qquad (12)$$

$$\frac{\rho}{\Lambda} = \frac{<\phi_1^*(x,0), \delta\nu\Sigma_1^f\phi_1(x,t)> + <\phi_2^*(x,0), \delta\nu\Sigma_2^f\phi_2(x,t)}{<\phi_2^*(x,0), \frac{1}{v_2}\phi_2(x,t)>}, \qquad (13)$$

where $< , >$ denotes integration over the entire reactor. The results of these computations are summarized in Figs. 1-3. Simple inspection of the figures indicates the possible difficulties inherent in point kinetics when they are used for fast transients and/or large reactors as discussed in section 2.1.

3. Stability Criteria for Nuclear Reactors

3.1 General Remarks

Even though point reactor kinetics may have limitations with regard to meaningful correlations of experimental results, they are quite appropriate to use for investigations of questions of stability or instability. Specifically, the neutron density in Eqs. (1-2) can always be written as the product of a time dependent function and a bounded shape function. With a convenient normalization of the shape function, the transport theory equations can always be reduced to a set of ordinary differential equations of the form of Eqs. (4-5), with or without the logarithmic term. The ordinary differential equations have coefficients ρ/Λ , β_{ie}/Λ etc which are bounded functions or functionals of the cross-sections and/or other reactor variables. Consequently, the problem of stability of nuclear reactors can be approached by assigning to the coefficients of the point kinetics equations a variety of bounded functional dependences and examining whether the time dependent solutions are stable or unstable. In addition, if by using the above procedure it is possible to establish criteria which: (a) guarantee stability; (b) can be evaluated experimentally and (c) are insensitive to small pertur-

bations of the physical constants of the system, then the question of physical interpretation of the time dependent variables, entering the point kinetics, loses its importance.

In order to present some general techniques which are, or may be used for stability studies, the kinetics equations are taken in the form of Eqs. (4-5) without the logarithmic and source terms. For simplicity and without loss of generality, it is assumed that the ratios β_{ie}/Λ are physical invariants of each reactor and that the ratio ρ/Λ can be expressed as a function or a functional of $G(t)$. In other words, the system of equations under consideration is:

$$\frac{dG(t)}{dt} = \left(\frac{\rho}{\Lambda} - \frac{\beta_e}{\Lambda}\right)G(t) + \sum_i^m \lambda_i C_i(t) \quad ; \tag{14}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{ie}}{\Lambda}G(t) - \lambda_i C_i(t) \; ; \tag{15}$$

$$\frac{\rho}{\Lambda} = \rho_e + \int_o^t f(t-\tau)\left[G(\tau)-G_o\right]d\tau + f_1(G(\tau<t)) \; , \tag{16}$$

where ρ_e is the externally controlled part of ρ/Λ , $f(t)$ is a kernel accounting for the linear dependence of ρ/Λ on $G(t)$, G_o is a constant representative of the operating power level and $f_1(G(\tau<t))$ is a functional such that ρ/Λ is bounded at all times and all values of $G(t)$.

An immediate property of Eqs. (14-16) which results from the boundedness of ρ/Λ is that no reactor admits a finite escape time. There are, however, some reactor models which yield solutions with finite escape times. [4]

3.2 Review of Existing Stability Criteria

For relatively small variations of ρ_e and $G(t)$, Eqs. (14-16) can be linearized and any of a number of linear stability criteria may be used to derive conditions that must be satisfied by $f(t)$ in order for the reactor to be stable. These conditions can be verified experimentally by means of oscillation or correlation tests. It must be recognized, however, that loss of linear stability does not necessarily imply that the solutions of the exact equations are unbounded. [5 - 6]

For larger variations of $G(t)$ and $f_1(G(\tau < t) \equiv 0$, a stability criterion has been derived by Welton without linearization of Eqs. (14-16). Specifically, the sufficient requirement for asymptotic stability is: [7 - 8]

$$\int_0^\infty f(t)\cos\omega t dt \leq 0 \qquad (17)$$

The meaning of this condition is that the reactor without delayed neutrons must be globally asymptotically stable or globally bounded, i.e. stable at all operating power levels (0 to ∞) and/or for all inputs ρ_e. The condition is very appealing because it relates nonlinear stability to experimentally detectable linear properties. It is, however, very restrictive and there is hardly any practical reactor that satisfies it.

Liapunov's second method has also been used to investigate the stability of Eqs. (14-16) with $f_1(G(\tau < t)) \equiv 0$. [9 - 11] It has been shown, however, that up to now the results of this approach are either equivalent to or more restrictive than Welton's criterion. [12 - 13]

3.3 Some Practical Considerations in Nonlinear Reactor Stability

From the preceding discussion, it follows that criteria for nonlinear reactor stability are not satisfactory. The purpose of this section is to discuss a number of changes which might be introduced in the approach to the problem in order to derive practical results.

(a) The importance of delayed neutrons In most reactor stability studies delayed neutrons are neglected. Thus the problem becomes easier and the results are always conservative provided that the feedback is not identically equal to zero. [14] If the feedback is equal to zero, then inclusion of delayed neutrons may deteriorate the reactor stability as in the case of sinusoidal oscillations of a critical reactor at zero power. [15]

From the practical standpoint, however, the omission of delayed neutrons is an oversimplification because the results maybe so conservative that they are impractical. [16] Consequently, delayed neutrons must be included in the analysis.

(b) The admissible operating power levels Many analyses of nonlinear reactor stability attempt to establish conditions for asymptotic stability at all operating power levels. Designing a reactor, however, to be globally asymptotically stable in most cases is unrealistic and unnecessary because of environmental and material constraints. It is more meaningful to derive stability criteria only for a practical and limited range of operating power levels.

(c) Asymptotic versus Lagrangian stability Requirements for asymptotic stability may be often so restrictive that they are impossible for the designer to achieve. In addition, even if a reactor is designed to be asymptotically stable, in practice the inherent noise leads to a mode of operation characterized by bounded variations which are not asymptotically stable. It is, therefore, of interest to examine other types of stability. For example, one possibility is to attempt to establish sufficient requirements for Lagrangian stability. /17/ Geometrically speaking, Lagrangian stability implies that the designer is satisfied with the assurance that all the reactor variables are confined within a closed region of the multidimensional state variable space instead of requiring that they tend to coalesce at one point. Lagrangian stability may be investigated either by means of Liapunov's second method /18/ or by examining the conditions under which the equilibrium states of the kinetics equations at infinity are locally unstable. Satisfaction of the latter conditions implies that the reactor variables are bounded or asymptotically stable depending on the particular topology of the equilibrium states in the finite space. /19/

3.3 Illustrative Examples

(a) A practical criterion for asymptotic stability in the small and Lagrange stability in the large Consider the stability of the solutions of Eqs. (14-16) with regard to step changes of ρ_e. Assume for simplicity that to each ρ_e = constant, there corresponds only one equilibrium state, G_1. If $G(t) = g(t) + G_1$, after some elementary algebra, the kinetics equations can be written as:

Table I

Summary of Data for Computer Experiment

Reactor	60 cm long	240 cm long
Material Composition ($\#/10^{-24}cm^3$)	Homogeneous mixtures of H, O, Zr, U^{235}, U^{238} $N_H = .30042 \times 10^{-1}$ $N_{Zr} = .10706 \times 10^{-1}$ $N_{28} = .69517 \times 10^{-4}$	$N_O = .24106 \times 10^{-1}$ $N_{25} = .92358 \times 10^{-3}$
Transverse Bucklings (cm^{-2})	3 material regions, 12 mesh points Region 1 ; points 0-3 ; $B^2 = .01091$ Region 2 ; points 3-9 ; $B^2 = .01200$ Region 3 ; points 9-12 ; $B^2 = .01089$	
Two Group Constants 1 = fast ; 2 = thermal	$D_1 = 1.69531$ $\Sigma_1^a = .137513 \times 10^{-1}$ $\Sigma_1^R = .164444 \times 10^{-1}$	$D_2 = .409718$ $\Sigma_2^a = .261361$
	$\nu\Sigma_1^f = .213143 \times 10^{-1}$ $\nu\Sigma_2^f = .544286$	$\nu\Sigma_1^f = .196962 \times 10^{-1}$ $\nu\Sigma_2^f = .497857$
Ramp Changes in Fission Cross-sections in Region 1 over 10 ms.	$\nu\Sigma_1^f$ varied from $.257143 \times 10^{-1}$ to $.169143 \times 10^{-1}$ $\nu\Sigma_2^f$ varied from .656644 to .431928	$\nu\Sigma_1^f$ varied from $.213561 \times 10^{-1}$ to $.176363 \times 10^{-1}$ $\nu\Sigma_2^f$ varied from .545353 to .450361

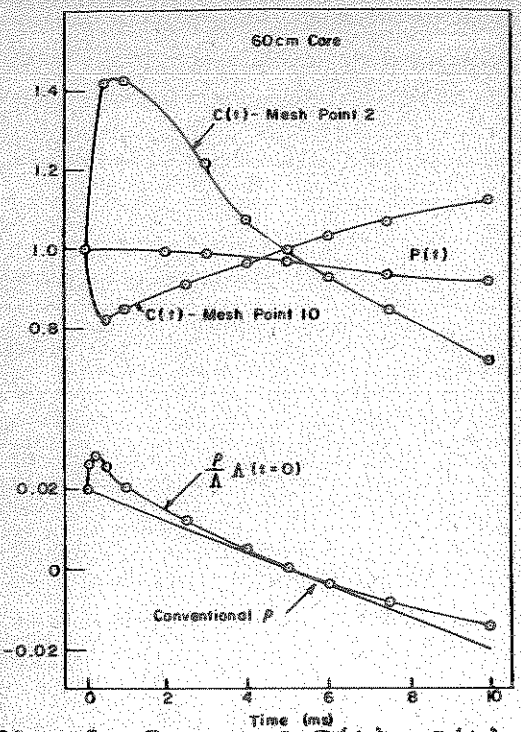

Fig. 1. Computed P(t), C(t) and $(\rho/\Lambda)(\Lambda(t=0))$ for 60 cm core

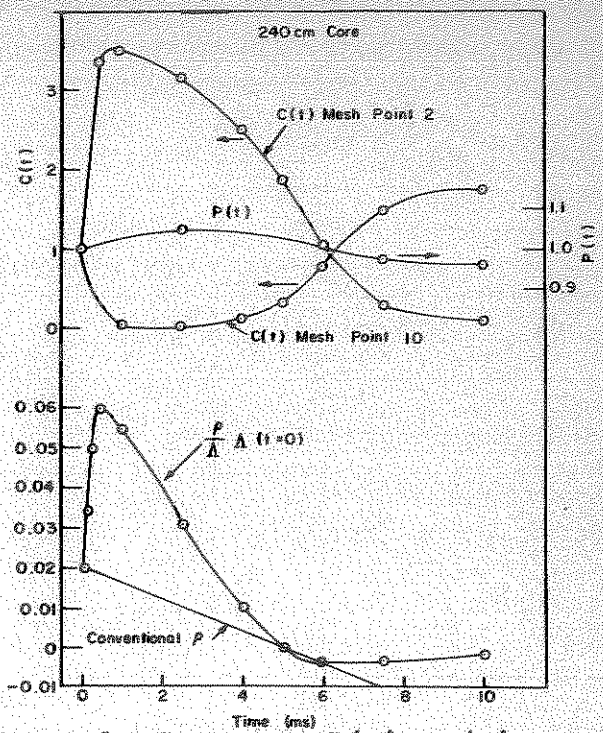

Fig. 2. Computed P(t), C(t) and $(\rho/\Lambda)(\Lambda(t=0))$ for 240 cm core

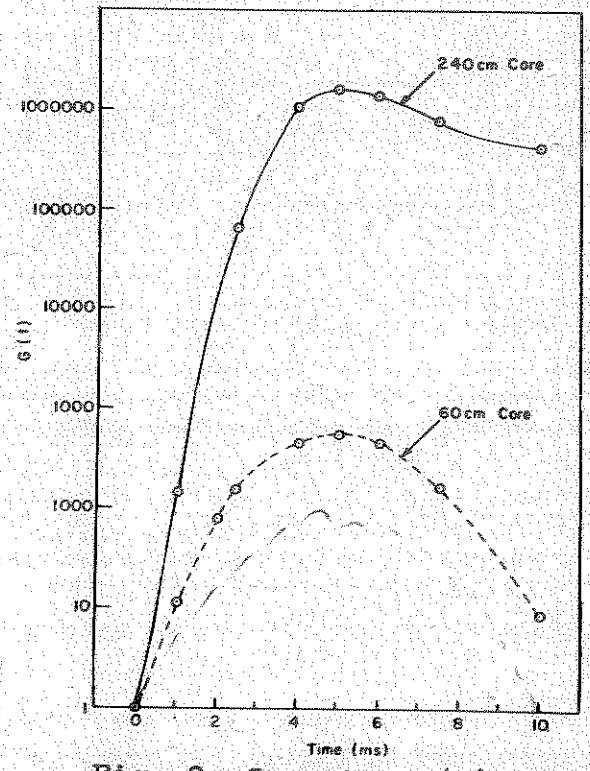

Fig. 3. Computed G(t) for 60 cm and 240 cm cores