ELECTRON TRANSPORT IN THREE-COMPONENT PLASMAS

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Abstract

A modified Chapman-Cowling approach is used to solve the linearized Boltzmann equation for electrons in a three-component plasma consisting of electrons, ions and neutrals. General expressions are obtained for the electron distribution function, current and heat flux. Specific values for the electron transport coefficients are reported for various collision models.

Introduction

This paper is part of a general study of particle and energy transport phenomena in thermionic converters under a variety of operating conditions. More specifically, the paper describes a novel technique for the analysis of electron transport phenomena. The technique is applicable to a variety of three-component, two temperature plasmas.

Transport phenomena in non-uniform gaseous mixtures have been analyzed by others (1-4). In particular, Chapman and Cowling (4) have developed a formalism for the solution of a set of Boltzmann equations through a series of successive approximations. This formalism has been successfully used in the field of gas dynamics where it is assumed that, to a first approximation, all components of the mixture have the same temperature.

In this paper a modified Chapman-Cowling approach is used to determine the electron distribution function, current and heat flux in a three-component plasma in which the electron temperature may be different from the ion and neutral particle temperatures. A similar method was recently presented by Stachanov and Stepanov (3). These authors, however, treated the charged particle collisions by means of the small angle Landau approximation and used a hard sphere model for electron-neutral collisions. In addition to removing these restrictions on the collision integrals, the present analysis yields the electron transport parameters in a form which is more amenable to physical interpretation.

The paper is divided into two parts. In Section 2, a perturbation method similar to that of Chapman and Cowling is used to solve the Boltzmann equation for electrons. In Section 3, the practicality of the perturbation method is illustrated both through its application to reference Lorentz plasmas, and through its application to a three-component plasma.

Perturbation Solution of the Boltzmann Equation

The Boltzmann Equations

The Boltzmann equations in a steady state, three-component plasma consisting of electrons (e), ions (i), and neutrals (n), may be written as

$$\bar{v}_\alpha \cdot \nabla_r f_\alpha + \frac{q_\alpha \bar{E}}{m_\alpha} \cdot \nabla_v f_\alpha = \sum_\beta J_{\alpha\beta}(f_\alpha, f_\beta) \quad ; \quad \alpha,\beta = e,i,n, \tag{1}$$

where $J_{\alpha\beta}$ is the collision integral for collisions between species α and β. This integral may be written in the Boltzmann form as:

$$J_{\alpha\beta}(f_\alpha, f_\beta) \equiv \iiint (f'_\alpha f'_\beta - f_\alpha f_\beta) g_{\alpha\beta} b \frac{db}{dx} dx d\epsilon d\underline{v}_\beta \tag{2}$$

where $g_{\alpha\beta} = |\bar{v}_\alpha - \bar{v}_\beta|$, b is the impact parameter, x and ϵ are the polar and azimuthal angles, respectively, describing the rotation (in center of mass coordinates) of the relative velocity vector during the collision, and $f'_\alpha = f_\alpha(\bar{r}, \bar{v}'_\alpha)$, where $\bar{v}'_\alpha$ is the velocity prior to the collision. In writing Eq. (2) for like-particle collisions, the second subscript is omitted. This eliminates confusion concerning the variables of integration. Further details concerning the geometry and derivation of Eq. (2) may be found in reference 5.

Small m_e/m_β Approximations

The system of Eqs. (1) represents, in general, a set of three-coupled, non-linear, six-dimensional equations for the distribution functions $f_\alpha(\bar{r}, \bar{v}_\alpha)$; $\alpha = e,i,n$. This system of equations may be greatly simplified by neglecting terms of order m_e/m_β ($\beta = i,n$) in the electron-heavy particle collision integrals:

$$J_{e\beta}(f_e, f_\beta) \simeq J_{e\beta}\left[f_e, n_\beta \delta(\bar{v}_\beta)\right],$$

$$J_{\beta e}(f_\beta, f_e) \simeq 0 \quad ; \quad \beta = i,n, \tag{3}$$

where n_β is the density of species β and $\delta(x)$ is the Dirac delta function. The physical implications of Eqs. (3) are two-fold: (a) in the electron Boltzmann equation the heavy particles may be treated as stationary scattering centers; (b) in the heavy particle Boltzmann equations, the electron-heavy particle collisions may be neglected entirely.

Use of approximation (3) in Eqs. (1) results in a system of two-coupled equations for the heavy particle distribution functions, plus an independent equation for the electron distribution function. The remainder of the paper is devoted to the solution of the electron equation. The solutions of the coupled heavy particle equations will be discussed in a future paper.

Linearized Boltzmann Equation

In seeking a solution for the electron distribution function $f_e(\bar{r},\bar{v}_e)$ it is convenient to define a perturbation function, $\phi_e(\bar{r},\bar{v}_e)$, by means of the equation:

$$f_e(\bar{r},\bar{v}_e) \equiv f_e^o(\bar{r},\bar{v}_e)\left[1 + \phi_e(\bar{r},\bar{v}_e)\right] , \tag{4}$$

where

$$f_e^o(\bar{r},\bar{v}_e) \equiv n_e(m_e/2\pi kT_e)^{3/2}\exp(- m_e v_e{}^2/2kT_e) . \tag{5}$$

$$n_e \equiv \int f_e d\underline{v}_e \quad ; \quad \tfrac{3}{2}n_e kT_e \equiv \int (m_e v_e^2/2) f_e d\underline{v}_e . \tag{6}$$

Thus, the perturbation ϕ_e must satisfy the conditions:

$$\int f_e^o \phi_e d\underline{v}_e = 0 \quad ; \quad \int v_e^2 f_e^o \phi_e d\underline{v}_e = 0 . \tag{7}$$

Under these definitions and conditions the linearized Boltzmann equation for electrons is:

$$\frac{1}{n_e} f_e^o \left\{ \bar{v}_e \cdot \left(\frac{\nabla_r P_e}{P_e} + \frac{e\bar{E}}{kT_e}\right) + (u_e^2 - \tfrac{5}{2})\bar{v}_e \cdot \frac{\nabla_r T_e}{T_e} \right\} =$$

$$= -n_e I_e(\phi_e) - n_i I_{ei}(\phi_e) - n_n I_{en}(\phi_e) \quad , \tag{8}$$

where

$$\bar{u}_e = (m_e/2kT_e)^{1/2}\bar{v}_e \tag{9}$$

and the linear integral operators I_e and $I_{e\beta}$ are defined by:

$$I_e[F_e(\bar{v}_e)] \equiv \frac{1}{n_e^2}\iiint f_e^o f^o (F_e + F - F_e' - F') g_e b\,db\,d\epsilon\,d\underline{v} , \tag{10}$$

$$I_{e\beta}\left[F_e(\bar{v}_e)\right] \equiv \frac{1}{n_e}\iint f_e^o (F_e - F_e') v_e b\,db\,d\epsilon \quad ; \quad \beta = i,n . \tag{11}$$

In the definitions (10) and (11), $F_e(\bar{v}_e)$ may be any scalar or vector function of $\bar{v}_e$ and $f^o = f_e^o(\bar{r},\bar{v})$, $F = F_e(\bar{v})$, $F_e' = F_e(\bar{v}_e')$ $F' = F_e(\bar{v}')$ and $g_e = |\bar{v}_e - \bar{v}|$. Also the operators I_e and $I_{e\beta}$ satisfy the symmetry relation (6):

$$\int H_e(\bar{v}_e) \cdot I\left[F_e(\bar{v}_e)\right] d\underline{v}_e = \int F_e(\bar{v}_e) \cdot I\left[H_e(\bar{v}_e)\right] d\underline{v}_e \quad ; \quad I = I_e, I_{e\beta} , \tag{12}$$

where $H_e(\bar{v}_e)$ and $F_e(\bar{v}_e)$ are arbitrary scalar or vector functions of $\bar{v}_e$.

Solution of the Linearized Boltzmann Equation

The general solution of Eq. (8) is:

$$\phi_e = -\frac{1}{n_e}(2kT_e/m_e)^{1/2}\left[\bar{A}_e(\bar{u}_e)\cdot\left(\frac{\nabla P_e}{P_e}+\frac{e\bar{E}}{kT_e}\right)+\bar{B}_e(\bar{u}_e)\cdot\frac{\nabla T_e}{T_e}\right]$$

$$+ C_1 m_e + C_2 m_e v_e^2/2 \ , \qquad (13)$$

where $\bar{A}_e$, $\bar{B}_e$ are vector functions of the electron velocity and C_1, C_2 are arbitrary constants. The first two terms on the right hand side of Eq. (13) represent the particular solutions corresponding to each of the driving terms in Eq. (8) while the last two terms represent the homogeneous solution.

Substitution of Eq. (13) into Eq. (8) yields:

$$f_e^o\bar{u}_e = n_e I_e(\bar{A}_e) + n_i I_{ei}(\bar{A}_e) + n_n I_{en}(\bar{A}_e) \ , \qquad (14)$$

$$(u_e^2 - \tfrac{5}{2})f_e^o\bar{u}_e = n_e I_e(\bar{B}_e) + n_i I_{ei}(\bar{B}_e) + n_n I_{en}(\bar{B}_e) \ . \qquad (15)$$

Since Eqs. (14) and (15) contain only $\bar{u}_e$ as an independent variable, $\bar{A}_e$ and $\bar{B}_e$ must be of the form:

$$\bar{A}_e(\bar{u}_e) = A_e(u_e)\bar{u}_e/u_e \quad ; \quad \bar{B}_e(\bar{u}_e) = B_e(u_e)\bar{u}_e/u_e \ , \qquad (16)$$

where $A_e(u_e)$ and $B_e(u_e)$ are scalar functions of the magnitude of $\bar{u}_e$.

From Eqs. (7) through (16) it is found that:

$$C_1 = C_2 = 0 \ , \qquad (17)$$

$$I_{e\beta}(\bar{G}) = \frac{1}{n_e} f_e^o v_e \sigma_{e\beta}(v_e)\bar{G} \quad ; \quad \bar{G} = \bar{A}_e, \bar{B}_e \ , \qquad (18)$$

where $\sigma_{e\beta}(v_e)$ is the momentum transfer cross section:

$$\sigma_{e\beta}(v_e) \equiv 2\pi\int(1-\cos x)b\,db \quad ; \quad \beta = i, n. \qquad (19)$$

The meaning of Eqs. (13) through (19) is that the problem of determining the electron distribution function $f_e(\bar{r},\bar{v}_e)$ is reduced to that of finding two scalar functions $A_e(u_e)$ and $B_e(u_e)$ which are solutions of Eqs. (14) and (15), respectively. These equations can be solved exactly only in the case of a Lorentz plasma (see Section 3). In the general case of a three-component plasma one resorts to approximation techniques as discussed below.

Sonine Polynomial Expansions for $A_e(u_e)$ and $B_e(u_e)$

In the case of a general three-component plasma, it is expedient to expand the scalar functions $A_e(u_e)/u_e$ and $B_e(u_e)/u_e$ into series of Sonine polynomials of order 3/2:

$$A_e(u_e)/u_e = \sum_{n=o}^{\infty} a_n S_n^{3/2}(u_e^2) \quad ; \quad B_e(u_e)/u_e = \sum_{n=o}^{\infty} b_n S_n^{3/2}(u_e^2) \ , \qquad (20)$$

where a_n, b_n are expansion coefficients and $S_n^{3/2}(x)$ is a Sonine polynomial of order 3/2. (7) Substitution of these expansions into Eqs. (14) and (15), dot-multiplication of the result by $S_m^{3/2}(u_e^2)\bar{u}_e$ and integration over velocity space yields two infinite sets of linear algebraic equations of the form:

$$\sum_{n=o}^{\infty} \alpha_{mn} a_n = \beta_m \quad ; \quad \sum_{n=o}^{\infty} \alpha_{mn} b_n = \gamma_m \quad ; \quad m = 0,1,\ldots\infty , \qquad (21)$$

where

$$\alpha_{mn} \equiv \alpha_{mn}^{ee} + \alpha_{mn}^{ei} + \alpha_{mn}^{en} \quad :$$

$$\alpha_{mn}^{ee} \equiv \int S_m^{3/2}(u_e^2)\bar{u}_e \cdot I_e\left[S_n^{3/2}(u_e^2)\bar{u}_e\right]d\underline{v}_e \quad ;$$

$$\alpha_{mn}^{e\beta} \equiv \frac{n_\beta}{n_e}\int S_m^{3/2}(u_e^2)\bar{u}_e \cdot I_{e\beta}\left[S_n^{3/2}(u_e^2)\bar{u}_e\right]d\underline{v}_e \quad ; \qquad (22)$$

$$\beta_m \equiv \frac{1}{n_e}\int S_m^{3/2}(u_e^2)u_e^2 f_e^o d\underline{v}_e = \frac{3}{2}\delta_{mo} \ ,$$

$$\gamma_m \equiv \frac{1}{n_e}\int S_m^{3/2}(u_e^2)u_e^2(u_e^2 - \frac{5}{2})f_e^o d\underline{v}_e = -\frac{15}{4}\delta_{m1} \ .$$

Thus, the problem of solving Eqs. (14) and (15) for the scalar functions $A_e(u_e)$ and $B_e(u_e)$ is reduced to that of solving the two infinite sets of Eqs. (21) for the Sonine expansion coefficients a_n and b_n. Approximate solutions to any desired degree of accuracy may be obtained by truncating the Sonine expansions after N terms and solving the resulting 2N equations. The matrix elements α_{mn} may in principle be determined once the collision laws are specified. More specifically, the quantities α_{mn}^{ee} are special cases of a general set of like-particle collision integrals which have been tabulated by Chapman and Cowling (6), while $\alpha_{mn}^{e\beta}$ can be evaluated by using Eq. (18) for the operator $I_{e\beta}$. From the symmetry relation of the I_e and $I_{e\beta}$ operators (Eq. 12), it is apparent that $\alpha_{mn} = \alpha_{nm}$.

Electron Current and Heat Flux

The electron, current, $\bar{J}_e$, and the electron heat flux $\bar{q}_e$, are:

$$\bar{J}_e \equiv e\int \bar{v}_e f_e d\underline{v}_e = -\mu_e \left\{ \nabla P_e + en_e\bar{E} + k_e^T n_e \nabla kT_e \right\} , \qquad (23)$$

$$\bar{q}_e \equiv \int \bar{v}_e \frac{m_e v_e^2}{2} f_e d\underline{v}_e = \frac{\bar{J}_e}{e} \frac{5}{2} kT_e + \frac{\bar{J}_e}{e} k_e^T kT_e - \mathscr{K}_e \nabla T_e \ , \tag{24}$$

where

$$\mu_e \equiv \frac{2e}{3n_e^2 m_e} \int f_e^o u_e A_e(u_e) d\underline{v}_e = \text{electron mobility} \tag{25}$$

$$k_e^T \equiv \frac{2e}{3n_e^2 m_e \mu_e} \int f_e^o u_e B_e(u_e) d\underline{v}_e = \text{thermal diffusion ratio} \tag{26}$$

$$\mathscr{K}_e \equiv \frac{\mu_e n_e k^2 T_e}{e} \left[\frac{2e}{3n_e^2 m_e \mu_e} \int f_e^o u_e^3 B_e(u_e) d\underline{v}_e - k_e^T (\tfrac{5}{2} + k_e^T) \right\} =$$

$$= \text{thermal conductivity.} \tag{27}$$

In deriving Eqs. (23-27), use is made of the equality:

$$\int f_e^o (u_e^2 - 5/2) \bar{A}_e \cdot \bar{u}_e d\underline{v}_e = \int f_e^o \bar{B}_e \cdot \bar{u}_e d\underline{v}_e \ . \tag{28}$$

The last term in Eq. (23) is the "thermal diffusion" term of Chapman and Cowling and other authors. The first term on the right hand side of Eq. (24) represents enthalpy transport, the second term is the "diffusion thermo-effect", an energy transport mechanism which is related to the thermal diffusion mechanism in Eq. (23) and the third term accounts for thermal conduction. The definitions of the transport coefficients μ_e, k_e^T and $\mathscr{K}_e$ are consistent with the usual definitions of these quantities. It should be noted that the thermal diffusion ratio appears as a transport coefficient in both Eqs. (23) and (24). This fact is a direct consequence of Eq. (28) and is in agreement with a general reciprocal theorem relating to the thermodynamics of irreversible processes. (9)

In terms of the Sonine expansion coefficients Eqs. (25-27) become:

$$\mu_e = \frac{ea_o}{n_e m_e} \ ; \quad k_e^T = \frac{b_o}{a_o} \ ; \quad \mathscr{K}_e = -\frac{5}{2} \frac{k^2 T_e}{m_e} (b_1 + \tfrac{2}{5} k_e^T b_o) \ . \tag{29}$$

Applications

Lorentz Plasmas

In the case of a Lorentz plasma Eqs. (14) and (15) may be solved exactly. Hence the Lorentz plasma provides a convenient reference case for comparisons. More specifically, for a Lorentz plasma consisting of electrons and heavy particles of species β only, the perturbation function and transport coefficients are:

$$\phi_e = - \lambda_{e\beta} \left[\frac{\nabla P_e}{P_e} + \frac{e\bar{E}}{kT_e} + (u_e^2 - \tfrac{5}{2}) \frac{\nabla T_e}{T_e} \right] \cdot \frac{\bar{u}_e}{u_e} \tag{30}$$

$$\mu_e = \frac{e}{P_e}\int\frac{v_e^2}{3\nu_{e\beta}} f_e^o d\underline{v}_e$$

$$k_e^T = \frac{e}{\mu_e P_e}\int\frac{v_e^2}{3\nu_{e\beta}}\left(\frac{m_e v_e^2}{2kT_e} - \frac{5}{2}\right) f_e^o d\underline{v}_e \tag{31}$$

$$\mathcal{H}_e = \frac{\mu_e k P_e}{e}\left[\frac{e}{\mu_e P_e}\int\frac{v_e^2}{3\nu_{e\beta}}\left(\frac{m_e v_e^2}{2kT_e}\right)^2 f_e^o d\underline{v}_e - \left(\frac{5}{2} + k_e^T\right)^2\right]$$

where $\lambda_{e\beta}(v_e) = 1/n_\beta\sigma_{e\beta}(v_e)$ is the electron mean free path and $\nu_{e\beta}(v_e) = v_e/\lambda_{e\beta}(v_e)$ is the e-β collision frequency. Note that the perturbation ϕ_e is small (compared to unity) when the fractional changes in the electron pressure and temperature and in the plasma potential are small over one mean free path.

Shown in Table 1 are values of the transport coefficients obtained from Eqs. (31) for three Lorentz plasmas of special interest: (a) the heavy particles are neutrals and ν_{en} = constant; (b) the heavy particles are neutrals and λ_{en} = constant; (c) the heavy particles are ions. A Lorentz plasma of the last type is primarily of mathematical interest since the neglect of e-e collisions as compared to e-i collisions requires that $n_e \ll n_i$, a condition which is never achieved in practice. Note that the values of Table 1 are in agreement with similar results derived by other techniques.

Table 1

Special Values of the Transport Coefficients for Lorentz Plasmas

Heavy Particle Species	Neutrals	Neutrals	Ions
Collision Law	ν_{en} = const	λ_{en} = const	Coulomb
μ_e	$\frac{e}{m_e\nu_{en}}$	$\frac{e\lambda_{en}v_a}{3kT_e}$	$128\left(\frac{\pi}{2}\right)^{1/2}\frac{\epsilon_o^2(kT_e)^{3/2}}{n_i e^3 m_e^{1/2}\ln\Lambda}$
k_e^T	0	-1/2	+3/2
$\mathcal{H}_e$	$\frac{5}{2}\frac{n_e k^2 T_e}{m_e\nu_{en}}$	$\frac{2}{3}n_e k\lambda_{en}v_a$	$512\left(\frac{\pi}{2}\right)^{1/2}\frac{n_e k\epsilon_o^2(kT_e)^{5/2}}{n_i e^4 m_e^{1/2}\ln\Lambda}$

Note: v_a = electron mean speed = $(8kT_e/\pi m_e)^{1/2}$
$\ln\Lambda$ = coulomb logarithm

The transport coefficients for a Lorentz plasma can also be computed approximately by means of the Sonine polynomial expansion technique. For example, for the case of a Lorentz plasma in which the heavy particles are neutrals and λ_{en} = constant, these coefficients for N = 1,2 and 3 can be readily evaluated and compared with the exact values given in Table 1. This comparison reveals (Table 2) that retention of three terms in each of the Sonine expansions is sufficient to yield all three transport coefficients correct to within 10%.

Table 2

Transport Coefficients for Lorentz Plasma with Constant λ_{en}

# of Terms in Sonine Expansion	$\mu_e/(\mu_e)$exact	$k_e^T/(k_e^T)$exact	$\mathcal{H}_e/(\mathcal{H}_e)$exact
N = 1	0.88	0	0
N = 2	0.95	0.77	0.85
N = 3	0.98	0.90	0.93

Three-Component Plasmas with Constant ν_{en}

Another illustration of the application of the Sonine polynomial expansion technique is the computation of the transport coefficients for a three-component plasma with ν_{en} = constant and $n_e \simeq n_i$. If three terms are again retained in each of the expansions, the matrix elements α_{mn} are readily evaluated and they lead to the following set of equations:

$$\begin{bmatrix} 1+1.50\,\nu_{en}/\nu_{ei} & 1.50 & 1.87 \\ 1.50 & 4.66+3.75\,\nu_{en}/\nu_{ei} & 5.37 \\ 1.87 & 5.37 & 10.7+13.1\,\nu_{en}/\nu_{ei} \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1.50 n_e/\nu_{ei} \\ 0 \\ 0 \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} 1+1.50\,\nu_{en}/\nu_{ei} & 1.50 & 1.87 \\ 1.50 & 4.66+3.75\,\nu_{en}/\nu_{ei} & 5.37 \\ 1.87 & 5.37 & 10.7+13.1\,\nu_{en}/\nu_{ei} \end{bmatrix} \begin{bmatrix} b_o \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.75 n_e/\nu_{ei} \\ 0 \end{bmatrix} \quad (33)$$

where $\nu_{ei} \equiv \dfrac{e^4 \ln\Lambda\, v_a n_i}{4\pi\epsilon_o^2 (2kT_e)^2}$ = effective electron-ion collision frequency.

Solution of Eqs. (32-33) yields a_n, b_n which in turn can be used in Eqs. (29) to calculate the transport coefficients. The results of the calculation are given in Table 3.

<u>Table 3</u>

<u>Transport Coefficients in a Three Component Plasma</u>

	Weakly Ionized Limit ($\nu_{ei} \ll \nu_{en}$)	Fully Ionized Limit ($\nu_{ei} \gg \nu_{en}$)	General Expression
μ_e	$\mu_e^o = \frac{e}{m_e \nu_{en}}$	$\mu_e^\infty = 0.57\times 128 (\frac{\pi}{2})^{1/2} \frac{\epsilon_o^2 (kT_e)^{3/2}}{e^3 n_i m_e^{1/2} \ln\Lambda}$	$\mu_e = \frac{\mu_e^o \mu_e^\infty}{\mu_e^o + \mu_e^\infty} h_\mu(\mu)$
k_e^T	$(k_e^T)^o = 0$	$(k_e^T)^\infty = 0.71$	$k_e^T = (k_e^T)^\infty h_k(\mu)$
$\mathcal{K}_e$	$\mathcal{K}_e^o = \frac{5}{2} \frac{n_e k^2 T_e}{m_e \nu_{en}}$	$\mathcal{K}_e^\infty = 0.23\times 512 (\frac{\pi}{2})^{1/2} \frac{k\epsilon_o^2 (kT_e)^{5/2}}{e^4 m_e^{1/2} \ln\Lambda}$	$\mathcal{K}_e = \frac{\mathcal{K}_e^o \mathcal{K}_e^\infty}{\mathcal{K}_e^o + \mathcal{K}_e^\infty} h_{\mathcal{K}}(\mu)$

The second column of Table 3 gives the values of the coefficients in the weakly ionized limit $\nu_{ei} \ll \nu_{en}$. They are denoted by the superscript "o" and are identical to the corresponding values given in Table 1. The reason is that the Sonine expansion technique with $N \geq 2$ yields an exact solution for the Lorentz plasma with constant ν_{en}.

The third column of Table 3 gives the values of the electron transport coefficients in the fully ionized limit $\nu_{ei} \gg \nu_{en}$. They are denoted by the superscript "∞" and are in excellent agreement with results reported in references 3 and 8. By comparing the values of μ_e^∞, $(k_e^T)^\infty$ and $\mathcal{K}_e^\infty$ with the corresponding values in Table 1 for a hypothetical e-i Lorentz plasma, the importance of electron-electron collisions is deduced. The effect of e-e collisions is to reduce each of the transport coefficients (in the fully ionized limit) to approximately 1/4-1/2 the value obtained in the absence of e-e collisions.

The fourth column of Table 3 gives expressions for the coefficients for values of ν_{ei}/ν_{en} which lie between the weakly ionized and fully ionized limits. The functions $h_\mu(\mu)$, $h_k(\mu)$ and $h_{\mathcal{K}}(\mu)$ are:

$$h_\mu(\mu) = \frac{1.00 + 7.00\mu + 9.67\mu^2 + 3.67\mu^3}{1.00 + 7.93\mu + 10.9\mu^2 + 3.67\mu^3} ,$$

$$h_k(\mu) = \frac{1.00\mu + 0.89\mu^2}{0.24 + 1.46\mu + 0.89\mu^2} , \qquad (34)$$

$$h_{\mathcal{K}}(\mu) = \frac{1.00 + 11.9\mu + 45.1\mu^2 + 73.2\mu^3 + 52.0\mu^4 + 13.5\mu^5}{1.00 + 13.9\mu + 62.0\mu^2 + 95.0\mu^3 + 65.9\mu^4 + 13.5\mu^5}$$

where $\mu \equiv \mu_e^o/\mu_e^\infty = 0.34\, \nu_{ei}/\nu_{en}$. These functions are plotted in Fig. (1).

From Fig. (1) it is apparent that the function $h_{\mu}(\mu)$ is nearly unity for all values of $\mu (0 \leq \mu \leq \infty)$. This implies that, to a good approximation, the contributions to the electron mobility from electron-neutral and from electron-charged particle collisions may be added in parallel. Similar conclusions apply to the contributions to the thermal conductivity of the electrons.

Conclusions

The modified Chapman-Cowling approach presented herein provides a useful tool for the analysis of three-component plasmas. The method yields both a quantitative description of the electron particle and energy transport mechanisms and an analytical expression for the electron distribution function.

Acknowledgement

The first author gratefully acknowledges a fellowship granted by Thompson Ramo Wooldridge Space Technology Laboratories.

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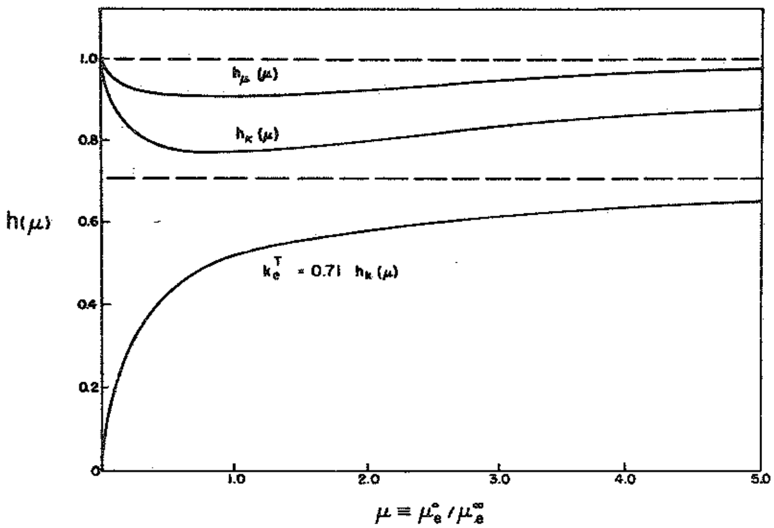

Figure 1. Plots of the $h(\mu)$ Functions vs μ