7

TRANSFER FUNCTION REPRESENTATION OF NUCLEAR POWER PLANTS

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Introduction

The concept of transfer function representation of nuclear power plants has proved a very fruitful tool both from the points of view of dynamic analysis and the interpretation of experimental data.

This fact can be best ascertained by considering the dynamics of a nuclear power plant, shown schematically in Fig. 1.
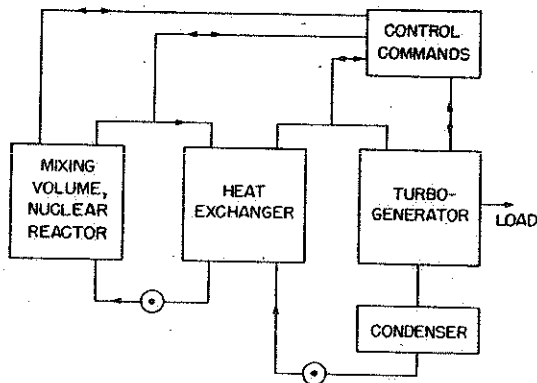

Fig. 1.

Schematic of Nuclear Power Plant.

Suppose all parameters of the plant have consistent equilibrium values prior to time $t = t_0$, when one or more of the parameters are perturbed intentionaly or unintentionally. Two questions arise:

1. Will the plant parameters seek equilibrium again, namely, is the system stable?

2. If the system is stable, what are the intermediate states its parameters assume between the two equilibria?

In order to find answers to those questions during the design phase of the nuclear plant, one starts by describing analytically the processes partaking in the dynamic operation of the plant. The principles of conservation of particles, mass, energy, momentum, etc., at all times are the basis of the formalism. The resulting equations relate the plant parameters, like neutron population, temperatures, pressures, flows, and load, to the independent variables: time and phase space. The equations are, generally, nonlinear, and/or linear, with variable coefficients, partial differential

equations. The solution of such a system of equations, which would answer the questions of stability and transient response, is extremely difficult if not impossible.[1] Consequently, one either resorts to machine-aided computations or one tries to simplify the analytical model of the plant without introducing excessive errors. The latter approach seems, to us anyway, more attractive.

The simplification of the analytical model is accomplished in two steps:

First, the phase-space dependence of the dynamic equations is integrated out, formally at least, by reformulating the problem in terms of appropriately defined derived variables,[2] like reactivity, neutron density amplitude, and average temperatures. The derived variables are customarily bilinear averages of the physical parameters of the plant and are related by ordinary differential equations with respect to time only. Those equations are still nonlinear and/or linear with variable coefficients.

Next, the ordinary differential equations are linearized by considering only small amplitude variations of the variables involved. Thus, the differential system can be reduced to a set of algebraic equations by the use of the Laplace transform technique. The algebraic equations can be solved explicitly to yield the desired relationship between any two variables of the plant, which are arbitrarily postulated as the input and output, respectively. This relationship is the transfer function.

Now, the questions of stability and transient response can be answered very elegantly by examining the characteristic modes of the transfer function and without ever finding the inverse Laplace transform. Bode, Nyquist or root-locus plots are very helpful in this respect. Furthermore, the characteristic modes of the transfer function are not related to the a priori postulated input and output of the system. Therefore, the arbitrary definitions of input and output do not limit the generality of the transfer function.

Apart from the fact that the use of the Laplace transformation leads to explicit solutions of systems of linear differential equations, it also affords a physical interpretation in terms of real frequencies. Such an interpretation makes possible the experimental verification of the implications derived from the analysis of transfer functions, as well as the experimental measurement of important parameters of the nuclear power plant. Oscillation tests, autocorrelation or cross-correlation techniques can be used very profitably for this purpose.

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[1] Cohen, E. R., Some Topics in Reactor Kinetics, Proc. Second Geneva Conference, 11, 302 (1958).

[2] Henry, A. F., Application of Reactor Kinetics to the Analysis of Experiments, Nuclear Sci. and Eng., 3, 52 (1958).

Of course, one might object that the concept of transfer function, no matter how elegant, is derived from such a simplified model of the plant dynamics that its implications are unreliable. This objection is quite valid, particularly when one is interested in the details of solutions of the dynamic equations for large amplitude variations of the variables involved. At the same time, though, the applicability of the concept of transfer function in nuclear power plants is not as restricted as the approximations introduced during its derivation tend to indicate. This assertion is based on the following facts:

1. The boundedness and stability of the solutions of a system of ordinary differential equations begins always with the examination of the type of stability at the equilibrium points of the differential system.[3,4] The equilibrium points are derived from the linearized equations and the type of stability is deduced from the characteristic modes which are identical with the characteristic modes of the transfer function.[5] Consequently, the concept of the transfer function is very important, even when the exact solution of nonlinear differential equations is investigated.

2. The nonlinear reactor equations are usually formulated in terms of the time-dependent amplitude of the first spatial mode of the neutron density. It can be shown[6] that the neglect of the modes of higher order introduces, particularly in cases of localized perturbations,[1] a larger error than the linearization of the dynamic equations. Therefore, the applicability of both the linear and the nonlinear approaches is limited.

3. The experimental data, which constitute the ultimate criterion of of any theory, are in very good agreement with the theoretical predictions derived from transfer function analysis.

4. The implications of the transfer function concept can be used very profitably even in cases where the physics of certain dynamic aspects of new nuclear plants is not well understood. Transfer function measurements allow one to explore the unknown effects safely during the development of the plant. The Boiling Water Reactor[7] is an example of this approach.

[3] Malkin, I. G., Certain Questions on the Theory of the Stability of Motion, Am. Math. Soc. Transl. No. 20, (1950).

[4] Liapunov, M. A., Problème Général de la Stabilité du Mouvement, Ann. Fac. Sci. Univ. Toulouse, 9 (2), 203-474 (1907).

[5] Smets, H. B. and E. P. Gyftopoulos, Topological Methods in Reactor Kinetics, Nuclear Sci. and Eng., 6, 341 (1959).

[6] Devooght, J., Nonlinear Reactor Dynamics, M. Sc. Thesis in Nuclear Eng. MIT, (June 1959).

[7] Kramer, A. W., Boiling Water Reactors, Addison-Wesley Publishing Co., Reading, Massachusetts (1958).

Components of Nuclear Power Plants

The major components of a nuclear power plant, in general, are: the nuclear reactor, the heat exchanger, the turbogenerator, and the condenser. Associated with those components are auxiliary devices, such as valves, pumps, and electronic controls (see Fig. 1).

The transfer function of each component is not universal. Its form depends on the structure, the mode of operation of the component, and the mental picture that one has about the sequence of events during dynamic operation.

The following sections present typical transfer functions of various components or processes of nuclear power plants and some conclusions that may be derived by means of simple manipulations.

Neutron Dynamics

The time-dependent equations for the neutron and delayed neutron precursors are

$$\frac{d}{dt} N(t) = \frac{\rho - \beta e}{\Lambda} N(t) + \sum_i^m \lambda_i C_i(t) + S(t) \tag{1}$$

$$\frac{d}{dt} C_i(t) = \frac{\beta_i e}{\Lambda} N(t) - \lambda_i C_i(t) \qquad (i = 1, 2 \ldots m). \tag{2}$$

Equations (1) and (2) can be derived either from a one-group model[8] or from general transport theory balance equations.[2] The neutron generation time, Λ,[9] rather than the neutron lifetime, ℓ, is used because, in most problems of practical interest, the reactor is perturbed through changes of the destruction rate. Therefore Λ, and not ℓ, is constant. Furthermore, when the concept of the generation time is used, the equations for the delayed neutron precursors do not contain the multiplication factor or reactivity.

If reactivity is assumed as the input to the reactor and neutron level amplitude as the output, two transfer functions can be derived from equations (1) and (2).

[8] Glasstone, S., Principles of Nuclear Reactor Engineering, D. Van Nostrand Co., New York (1955).

[9] Lewins, J., The Use of the Generation Time in Reactor Kinetics, Nuclear Sci. and Eng., 7, 122 (1960).

1. Transfer Function of Critical Reactor

The source $S(t)$ is zero. The linearization of equations (1) and (2) and transformation of the time variable into frequency yields:

$$\frac{\overline{N}}{\overline{\rho}} = \frac{N_0}{s\left[\Lambda + \sum_i^m \frac{\beta_{ie}}{s + \lambda_i}\right]} . \tag{3}$$

The same transfer function has been derived by various authors after introducing certain approximations in the basic kinetics equations.[10] The approximations are not necessary in the present derivation because of the use of the concept of the generation time.

2. Transfer Function of Subcritical Reactor

The source $S(t) = S_0 = $ constant. The transfer function is

$$\frac{\overline{N}}{\overline{\rho}} = \frac{N_0}{\Lambda s + \sum_i^m \frac{\beta_{ie} s}{s + \lambda_i} + \Lambda S_0} . \tag{4}$$

It is evident from equation (4) that a subcritical reactor does not act as an integrator, as when $S_0 = 0$. This fact is important when interpreting experimental data from reactors with built-in sources, like the heavy water reactors.

Equivalent Delayed Neutron Groups

The transfer function of the reactor provides the means for approximating the m delayed neutron precursors by $k(<m)$ equivalent groups.

The basic idea is to evaluate the constants of the equivalent delayed neutron groups (λ_j^*, β_j^*) so that the transfer functions, of the exact and approximate systems, are almost identical over broad ranges of frequency. This idea may be implemented by means of the method of moments approximation.

More specifically, the approximate transfer function is

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[10]Soodak, H. and E. C. Campbell, Elementary Pile Theory, John Wiley & Sons, Inc., New York (1950).

$$\frac{\overline{N}}{\overline{\rho}} = \frac{N_0}{s\left[\Lambda + \sum_j^k \frac{\beta_j^*}{s + \lambda_j^*}\right]}$$
(5)

If a high-frequency approximation is desired, the functions (3) and (5) may be expanded into power series around $1/s = 0$ and the coefficients of the first $(2k)$ powers of $1/s$ taken as equal. Thus $(2k)$ equations result:

$$\sum_j^k \beta_j^* \lambda_j^{*\,q} = \sum_i^m \beta_{ie} \lambda_i^{\,q} \qquad [q = 0,1...(2k-1)],$$
(6)

from which λ_j^* and β_j^* can be determined. A similar procedure can be followed for small frequencies but then the power series expansions should be performed around $s = 0$. Thus it is found that

$$\sum_j^k \beta_j^* \lambda_j^{*-q} = \sum_i^m \beta_{ie} \lambda_i^{-q} \qquad [q = 1,2...2k].$$
(7)

Furthermore, the large and small-frequency approximations can be combined to yield a better overall approximation of the actual transfer function. The 2 k equations that must be solved are

$$\left.\begin{array}{l} \displaystyle\sum_j^k \beta_j^* \lambda_j^{*\,q} = \sum_i^m \beta_{ie} \lambda_i^{\,q} \qquad [q = 0,1...(k-1)] \\[2em] \displaystyle\sum_j^k \beta_j^* \lambda_j^{*-p} = \sum_i^m \beta_{ie} \lambda_i^{-p} \qquad [p = 1,2...k] \end{array}\right\}$$
(8)

It should be pointed out that the small-frequency approximation is good for very slow transients while the large frequency approximation is valid for fast transients.

Reactor Core Thermal Dynamics

The reactor core thermal dynamics refers to the relationships between the power, coolant input temperature, and coolant flow, and the temperature distribution throughout the reactor core. It is also related to the effects of the temperature distribution on criticality or reactivity.

The thermal dynamics transfer functions are not universal. They depend on the particular reactor in question and the conceptual model that one forms about the sequence of events involved.

The presentation of all the models that have been derived for different reactors is beyond the scope of this paper. The interested reader is referred to references 11 through 17. In order to discuss the methodology of the development of thermal dynamics transfer functions, only a distributed and a lumped parameter model are given.

Transfer Functions of a Distributed Parameter Reactor Core

Consider a heterogeneous reactor with fuel, moderator and one-phase coolant regions. The heat exchange is mostly one-dimensional from the fuel region perpendicularly to the coolant, through the moderator. The reactor is assumed as made of groups of three slabs (see Fig. 2), without any loss of generality, because such a geometry can be adapted to many reactor designs.[18]
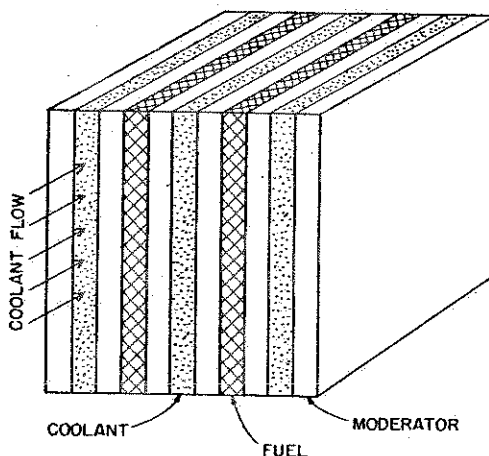
Fig. 2.
Reactor Core Model

[11] Schultz, M. A., Control of Nuclear Reactors and Power Plants, McGraw-Hill Book Co., New York (1955).

[12] MacPhee, J., The Stability of Multipass Reactors, Nuclear Sci. and Eng. 4, 200 (1958); The Kinetics of Circulating Fuel Reactors, Nuclear Sci. and Eng. 4, 588 (1958).

[13] Skinner, R. E. and D. L. Hetrick, The Transfer Function of a Water Boiler Reactor, Nuclear Sci. and Eng. 3, 573 (1958).

[14] Remley, M. E. et al, Kinetic Experiments on Water Boiler Reactors, Proc. Second Geneva Conf. 11, 447 (1958).

[15] Beckjord, E. S., Dynamic Analysis of Boiling Water Reactors, ANL-5799 (1958).

[16] Ackroyd, R. T., Stability Considerations in the Design of Fast Reactors, Proc. Second Geneva Conf. 12, 230 (1958).

[17] O'Neill, T. J., Heat Transfer Transient Equations for Graphite Reactors, Proc. Second Geneva Conf. 11, 269 (1958).

[18] Larson, C. L., Heat Transfer in a Cross Flow Nuclear Reactor, Nuclear Sci. and Eng. 4, 607 (1958).

Assume that the inputs to the reactor core are power, coolant velocity and coolant input temperature. Consider two temperature coefficients of reactivity. One is associated with the mean fuel temperature and another with the mean moderator temperature. Under those conditions the transfer functions of interest are the ones shown schematically in Fig. 3.
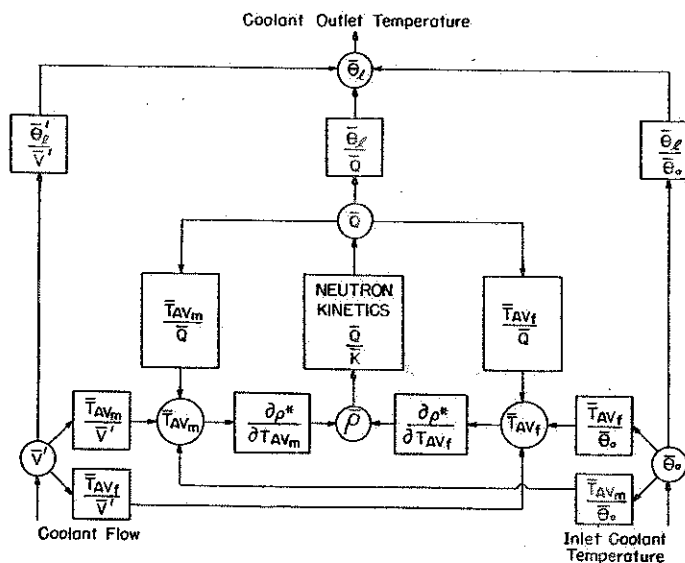

Fig. 3. Reactor Core Block Diagram.

The transfer functions are derived from the space and time-dependent partial differential equations. It is assumed that:

1. The heat is released in the fuel slabs and carried away by the nearest coolant slabs.

2. The heat is uniform throughout the reactor core.

3. The heat is transferred by conduction in the fuel and moderator regions and by convection in the coolant region.

4. The temperature across the coolant region is constant, because of turbulence of the fluid.

5. The cross section of the coolant slabs is varied so that the outlet coolant temperature is equal for any slab.

6. The heat transfer coefficient h is proportional to $v^{0.8}$.

The details of the derivation of the transfer functions are given in reference 19. It is found that:

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[19] Gyftopoulos, E. P. and H. B. Smets, <u>Nuclear Power Plant Transfer Functions,</u> Nuclear Eng. and Sci. Conf., Preprint 53; Chicago (1958) <u>Distributed Parameter Reactors</u>, Nuclear Sci. and Eng. <u>5</u>, 405 (1959).

$$\frac{\overline{\theta}_\ell}{\overline{Q}} = \frac{y_{12}}{h \cos h(qy_0)} \left[\frac{s+b}{vf_1} - 1 \right] \left[1 - e^{-f_1\ell} \right] \qquad (9)$$

$$\overline{\theta}_\ell / \overline{\theta}_0 = e^{-f_1\ell} \qquad (10)$$

$$\frac{\overline{\theta}_\ell}{\overline{v}'} = \frac{Q_0 y_{12}}{v^2 h f_1} \left[0.8 (s - vf_1) + b \right] \left[1 - e^{-f_1\ell} \right] \qquad (11)$$

$$\frac{\overline{T}_{AVm}}{\overline{Q}} = y_{12} \left[\frac{\tanh(qy_0)}{hqy_0 \left\{ 1 + \frac{kq}{h} \tanh(qy_0) \right\} \cosh(qy_0)} \right.$$

$$\left. \cdot \left\{ \frac{1 - e^{-f_1\ell}}{f_1\ell} + \frac{s+b}{vf_1} \left[1 - \frac{1 - e^{-f_1\ell}}{f_1\ell} \right] \right\} + \frac{\cosh(qy_0) - 1}{kq^2 y_0 \cosh(qy_0)} \right]$$

$$(12)$$

$$\frac{\overline{T}_{AVm}}{\overline{\theta}_0} = \frac{\tanh(qy_0)}{f_1\ell \, qy_0 \left[1 + \frac{kq}{h} \tanh(qy_0) \right]} \left[1 - e^{-f_1\ell} \right] \qquad (13)$$

$$\frac{\overline{T}_{AVm}}{\overline{v}'} = \frac{Q_0 y_{12} \tanh(qy_0)}{hqy_0 v} \cdot \frac{\dfrac{1 - e^{-f_1\ell}}{f_1\ell} \left[0.8 - \dfrac{0.8s + b}{vf_1} \right] + \dfrac{0.8s + b}{vf_1}}{1 + \dfrac{kq}{h} \tanh(qy_0)}$$

$$(14)$$

$$\frac{\overline{T}_{AVf}}{\overline{Q}} = \frac{vb}{\ell q_2^2 y_{12}} \cdot \frac{\ell \frac{\Lambda}{\pi} - 1 + e^{-\frac{\Lambda}{\pi}\ell}}{\Lambda^2 h} \sinh^2(q_2 y_{12}) + \frac{\alpha_2}{k_2 s} \qquad (15)$$

$$\frac{\overline{T}_{AVf}}{\overline{\theta}_0} = \frac{v}{\ell y_{12}} \left[\frac{1 - e^{-\frac{\Lambda}{\pi}\ell}}{q_2 \Lambda} \sinh(q_2 y_{12}) \right] \qquad (16)$$

$$\frac{\overline{T}_{AVf}}{\overline{v}'} = \frac{Q_0 \sinh(q_2 y_{12})}{hq_2} \left[0.8 \frac{1 - e^{-\frac{\Lambda}{\pi}\ell}}{\Lambda\ell} + \frac{0.8s + b}{v\Lambda} \left(1 - \frac{1 - e^{-\frac{\Lambda}{\pi}\ell}}{\Lambda\ell} \pi \right) \right],$$

$$(17)$$

where

$$f_1 = \frac{s}{v} + \frac{b}{v} \cdot \frac{\frac{kq}{h}\tanh(qy_0)}{1 + \frac{kq}{h}\tanh(qy_0)}$$

$$\Lambda = s\frac{q_2 k_2}{q_1 k_1}\sinh(q_1 y_1) \cdot \sinh(q_2 y_{12}) + (s+b)\frac{q_1 k_1}{h}\sinh(q_1 y_1) \cdot \cosh(q_2 y_{12})$$

$$+ \; s\cosh(q_1 y_1) \cdot \cosh(q_2 y_{12}) + (s+b)\frac{q_2 k_2}{h}\cosh(q_1 y_1) \cdot \sinh(q_2 y_{12})$$

$$\pi = v\frac{q_2 k_2}{q_1 k_1}\sinh(q_1 y_1) \cdot \sinh(q_2 y_{12}) + v\frac{q_1 k_1}{h}\sinh(q_1 y_1) \cdot \cosh(q_2 y_{12})$$

$$+ \; v\cosh(q_1 y_1) \cdot \cosh(q_2 y_{12}) + v\frac{q_2 k_2}{h}\cosh(q_1 y_1) \cdot \sinh(q_2 y_{12})$$

$$q_i^2 = s/\alpha_i \quad .$$

All the other symbols are defined in the nomenclature.

The transfer functions are transcendental and very cumbersome to use, which is typical of distributed parameter systems. However, both graphical[19] and analytical[20,21] methods have been devised for their approximation in terms of rational functions and pure delays. Two typical approximations are

$$\frac{\bar{\theta}_\ell}{\bar{\theta}_0} = e^{-\frac{b\ell}{v}} + \frac{(1 - e^{-\frac{b\ell}{v}})(1 + \tau_1 s)}{(1 + \tau_2 s)(1 + \tau_3 s)} e^{-\frac{s\ell}{v}} \tag{18}$$

$$\frac{\bar{\theta}_\ell}{\bar{Q}} \propto \frac{1}{1 + \tau_4 s}\frac{1 - \tau_5 s}{1 + \tau_5 s} \quad . \tag{19}$$

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[20] Sandmeier, A. H., Time Constant of Cylindrical Fuel Pin, Nuclear Sci. and Eng., 5, 186 (1959).

[21] Iriarte, M., Heat Release in Cylindrical Fuel Elements, Nuclear Sci. and Eng., 7, 26 (1960).

The time constants τ_i are complicated, implicit functions of the geometry and the steady-state parameters of the reactor. They can be found by trial-and-error matching of the exact functions, over a broad frequency range, along the $s = j\omega$ axis. Particular examples of the exact and approximate transfer functions $\overline{\theta}_\ell / \overline{\theta}_0$ and $\overline{\theta}_\ell / \overline{Q}$, for $s = j\omega$, are given in Figs. 4 and 5, respectively.
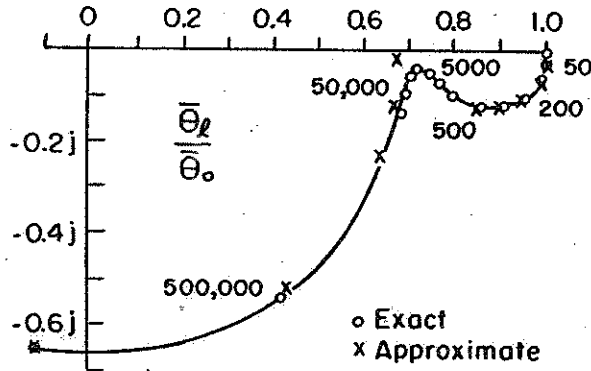
Fig. 4.
Exact and Approximate Temperature Transfer Functions for $s = j\omega$ (ω in hr^{-1}).

Fig. 5.
Exact and Approximate Power Transfer Functions for $s = j\omega$ (ω in hr^{-1}).
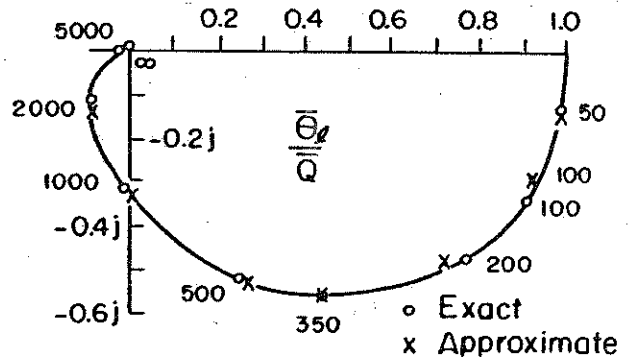
Transfer Functions of a Lumped Parameter Reactor Core

Consider a reactor with two temperature coefficients of reactivity, r_1 and r_2, related to two representative core temperatures, T_1 and T_2, respectively. Assume a model independent of spatial coordinates and whose thermal dynamics is describable by Newton's law of cooling, namely,

$$\frac{dT_1}{dt} = a_1 N - g_1 T_1 \tag{20}$$

and

$$\frac{dT_2}{dt} = a_2 N - g_2 T_2 \ . \tag{21}$$

Both the production coefficients a_i and the heat removal coefficients g_i are constant. The block diagram of this reactor is shown in Fig. 6, where

$$\frac{\overline{T}_1}{\overline{N}} = \frac{a_1}{s + g_1}; \quad \frac{\overline{T}_2}{\overline{N}} = \frac{a_2}{s + g_2}; \quad H(s) = r_1\frac{\overline{T}_1}{\overline{N}} + r_2\frac{\overline{T}_2}{\overline{N}} \quad . \tag{22}$$
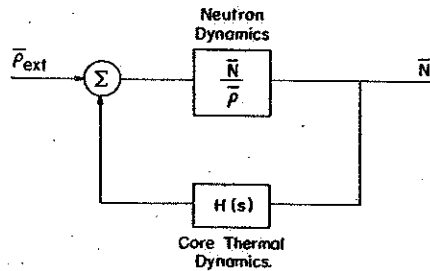
Fig. 6.
Reactor Core Reduced Block Diagram.

It is interesting to discuss the stability of the reactor for various values of the temperature coefficients r_1 and r_2. For the purposes of this discussion the effects of the delayed neutron precursors are neglected from the neutron transfer function, which reduces to

$$\frac{\overline{N}}{\overline{\rho}} = \frac{N_0}{\Lambda s} \quad . \tag{23}$$

The application of Nyquist's or root-locus criterion of stability reveals the following:

    1.   If both temperature coefficients are positive $(r_1 > 0, r_2 > 0)$, the reactor is obviously always unstable.

    2.   If both temperature coefficients are negative $(r_1 < 0, r_2 < 0)$, the reactor is always stable.

    3.   If one temperature coefficient is positive and one is negative $(r_1 > 0, r_2 < 0)$, then, when

$$r_1 a_1 g_1 + r_2 a_2 g_2 < 0; \qquad\qquad r_1 a_1 g_2 + r_2 a_2 g_1 < 0 \tag{24}$$

the reactor is always stable. However, if

$$r_1 a_1 g_1 + r_2 a_2 g_2 > 0; \qquad\qquad r_1 a_1 g_2 + r_2 a_2 g_1 < 0, \tag{25}$$

which implies that $g_1 > g_2$, the reactor is stable only when the neutron density is

$$N_0 < \frac{\Lambda\, g_1 g_2 (g_1 + g_2)}{r_1 a_1 g_1 + r_2 a_2 g_2} \quad . \tag{26}$$

Those results are summarized in graphical form in Fig. 7. The graphs clearly indicate the well-known fact that the importance of a temperature coefficient is determined by its promptness or delayedness which, in the present case, is the inverse heat removal coefficient.
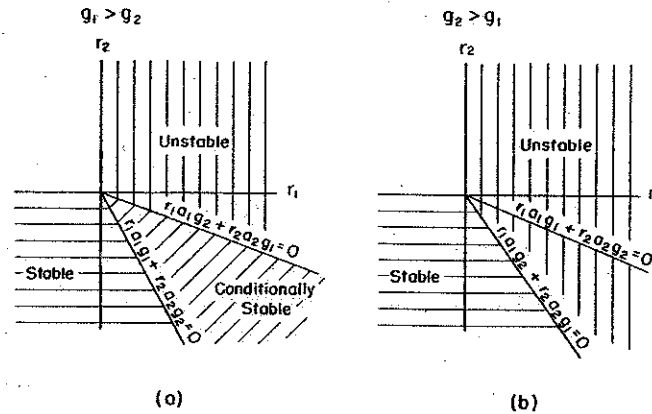

(a)   (b)

Fig. 7. Regions of Stability of Reactor with Two
          Temperature Coefficients. Lumped
          Parameter Model.

Stabilizing Effects of Heat of Radioactive Decay

Part of the heat of radioactive decay of fission products is transferred directly to the coolant. This fact is usually omitted from the analytical model of the thermal dynamics, even though it has a favorable influence on the stability of the reactor.

Consider a water reactor with plate-type fuel elements. Assume a lumped parameter model with one temperature coefficient associated with the coolant. Neglect the heat exchanger coupling.

If all the heat of fission is produced in the fuel, the feedback transfer function is[11]

$$H(s) = \frac{k}{s^2 + \left[\dfrac{1}{\tau_1} + \dfrac{1}{\tau_2} + \dfrac{2}{\tau_0}\right] s + \dfrac{2}{\tau_1 \tau_0}} \quad . \tag{27}$$

The root locus of the characteristic modes of the overall transfer function is as shown in Fig. 8. Evidently, the reactor becomes unstable above a certain power level indicated by the crossover points (A, A') of the locus.

Now, if a fraction α of the total heat is delivered directly to the coolant, it can be easily proved that the feedback transfer function is

$$H(s) = \frac{k\alpha\tau_1\left(s + \frac{1}{\alpha\tau_1}\right)}{s^2 + \left[\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{2}{\tau_0}\right]s + \frac{2}{\tau_1\tau_0}} \quad . \tag{28}$$

The reactor is stable for all power levels (Fig. 9), provided that

$$\alpha > \frac{1}{1 + \frac{\tau_1}{\tau_2} + 2\frac{\tau_1}{\tau_0} + \sum_i^m \frac{\beta_{ie}}{\Lambda}\left[\sum_j^m \lambda_j - \lambda_i\right]\tau_1} = \alpha_0 \tag{29}$$

For $\Lambda \leq 10^{-4}$, condition (29) yields $\alpha_0 \leq 1.2\%$, and $\alpha > \alpha_0$ is satisfied by many reactors. Therefore, it is ascertained that the heat of radioactive decay has a stabilizing influence.
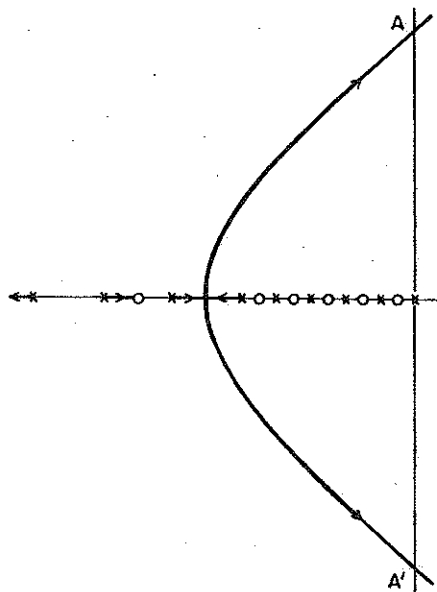

Fig. 8. Root Locus of Characteristic Modes of Heterogeneous Reactor with One Temperature Coefficient. All Heat is Released in the Fuel.
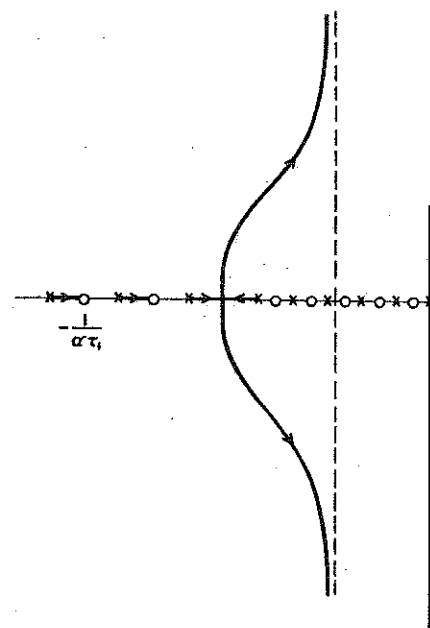
Fig. 9. Root Locus of Characteristic Modes of Heterogeneous Reactor with One Temperature Coefficient. A Fraction of Heat (α) is Transferred Directly to the Coolant.

Effects of Delayed Neutrons on Stability

It has been shown, quite rigorously,[22] that the delayed neutron precursors influence favorably the stability of a reactor. This fact can also be illustrated by means of the transfer function concept as follows:

Consider, as an example, the case of the EBWR. The block diagram of the reactor is shown in Fig. 6,[7] where

$$H(s) = \frac{k_1(1 + \tau_1 s)}{(1 + \tau_2 s)(1 + \tau_3 s)(1 + \tau_4 s)(1 + \tau_5 s)} \tag{30}$$

Suppose that the delayed neutron precursors are omitted from the neutron transfer function. The root locus of the characteristic modes of the overall transfer function is then as shown in Fig. 10. The power level at which instability occurs is, to a very good approximation, proportional to the ratio of the distances of the crossover point A from the poles and zeros (stars and noughts) of the locus, respectively.
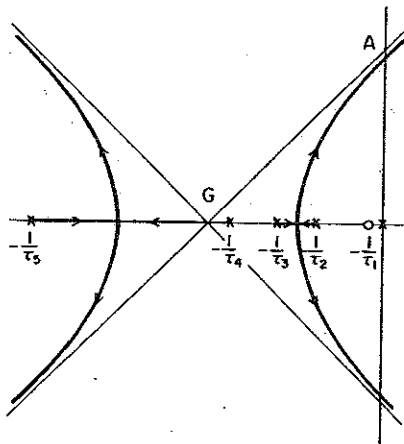

Fig. 10
Root Locus of Characteristic Modes of
EBWR without Delayed Neutrons.

Now, suppose the delayed neutrons are included in the neutron transfer function and examine their effects on the root locus. It is evident that the asymptotic behavior of the locus will not be affected because m negative poles and m negative zeros have been introduced (see Fig. 11). However, the center of gravity G (Fig. 10) of the pole-zero configuration has been shifted to G' because the difference between the poles and zeros introduced by the delayed neutrons is

[22] Gyftopoulos, E. P. and J. Devooght, Effects of Delayed Neutrons on Nonlinear Reactor Stability, to be published.

$$\sum_i^m (\text{poles}) - \sum_i^m (\text{zeros}) = -\left[\sum_i^m \lambda_i + \sum_i^m \frac{\beta_{ie}}{\Lambda}\left(\sum_j^m \lambda_j - \lambda_i\right)\right] + \sum_i^m \lambda_i$$

$$= -\sum_i^m \frac{\beta_{ie}}{\Lambda}\left(\sum_j^m \lambda_j - \lambda_i\right) < 0 \quad . \tag{31}$$

This shift changes the crossover point from A to A' and simple inspection and comparison of the loci, with and without delayed neutrons, reveals immediately that the new power level for instability is higher. Hence, the delayed neutrons have a stabilizing influence on the reactor.
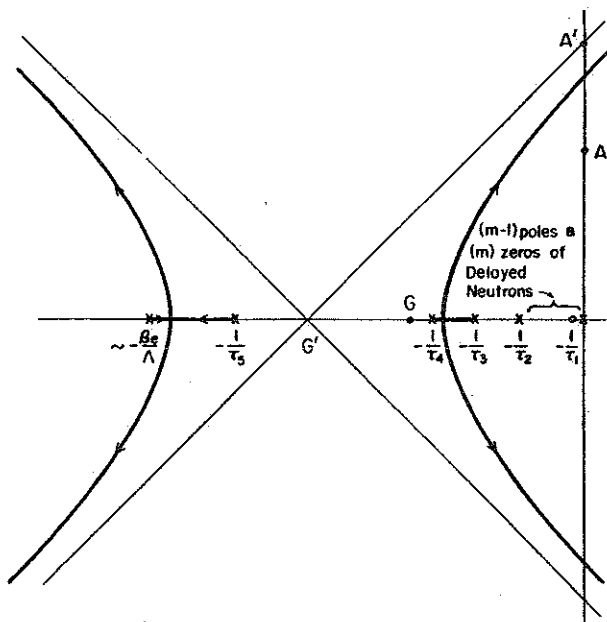

Fig. 11
Root Locus of Characteristic Modes of EBWR with Delayed Neutrons.

It should be pointed out that the use of the EBWR is incidental and the previous procedure and conclusions are applicable to any reactor model.

Heat Exchanger Dynamics

The dynamics of heat exchangers is extremely involved both from the physical and the analytical standpoints. In spite of the fact that heat exchangers have been in use for many decades, it is only recently that the problem of their dynamic operation has attracted the attention of various authors. A complete discussion of recent developments and an extensive bibliography can be found in reference 23.

[23] Hansen, P. D., The Dynamics of Heat Exchange Process, ScD. Thesis, M. Eng. MIT, (Feb. 1960).

Transfer functions of heat exchangers have appeared in the literature.[11,24-26] Both lumped and distributed parameter models have been considered, and the transfer functions have been derived under a variety of assumptions. Unfortunately, there is practically no experimental information available with which the analytical models can be evaluated. This is particularly true for variable flow conditions.

In what follows, the transfer functions of a counter flow heat exchanger (Fig. 12) are presented. These functions were derived[19] from a distributed parameter, tube-and-shell model (see Fig. 13), under the following assumptions:

1. The primary coolant does not change phase.

2. The flow is turbulent in both fluids.

3. The heat conduction along the axial direction of the tube and the shell is zero.

4. The heat conductivity along the radius of the tube and the shell is infinite.

5. The heat losses to the surroundings are zero.

6. The heat transfer coefficients are independent of temperature for each region.

7. The boundaries between economizer-boiler and boiler-superheater are fixed.
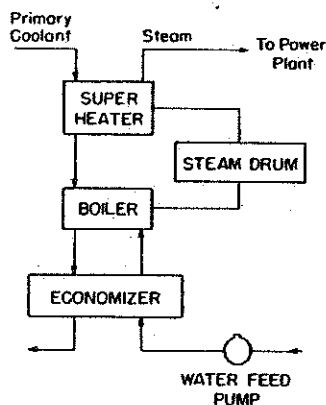

Fig. 12.
Counter Flow Heat Exchanger.

[24] Paynter, H. M. and Y. Takahashi, Counterflow and Parallel Flow Heat Exchangers, Trans. ASME 749, (May 1956).

[25] Takahashi, Y., Transfer Function of Heat Exchange Processes, Automatic and Manual Control, Butterworth Scientific Publications (1952).

[26] Sutton, R. W., A Study of the Transient Behavior of Heat Exchangers, AERE-LEO/A 63 (1953).
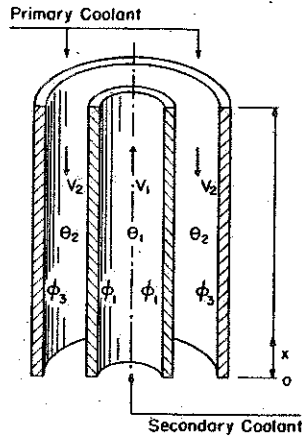
Fig. 13.
Heat Exchanger Model.

The derived transfer functions are shown in Figs. 14 and 15. The details of the derivations can be found in reference 19.
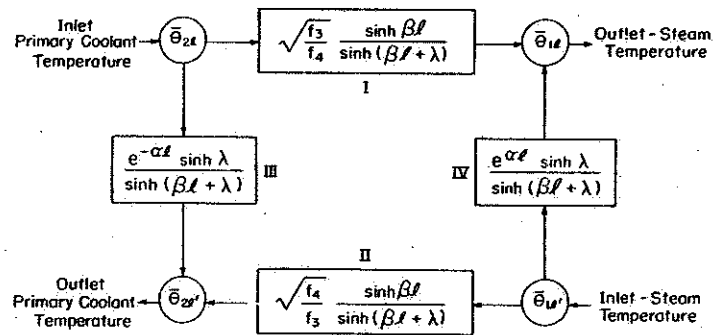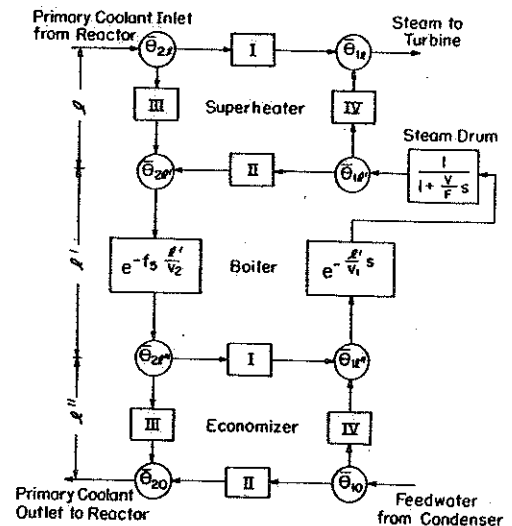

Fig. 14. Superheater Transfer Functions.
Distributed Parameter Model.

Fig. 15.
Counter Flow Heat Exchanger Transfer
Functions. Distributed Parameter
Model.

Again, the transfer functions are transcendental but can be approximated by rational functions and pure delays.

It should be pointed out that the dynamic behavior of heat exchangers is due to thermal capacities and mass transports. Therefore, the open loop response to flow or temperature changes is monotonic and non-oscillatory. However, in natural-circulation circuits, stable and unstable oscillations may occur due to couplings of thermal and mechanical effects.

Turbogenerator Dynamics

The dynamics of turbogenerators is also very difficult to describe and not so well understood. The problem is complicated because it is related to transient thermodynamics, a field which has not been fully explored yet.[27]

Approximate transfer functions have been derived and some typical examples are subsequently presented.

Consider the turbogenerator shown schematically in Fig. 16. Transfer functions for the unit are shown in Fig. 17. They are derived under the following assumptions:

1. The pressure in front of the throttling valve is constant.

2. The turbine angular speed is regulated by a speed controller.

3. The alternator has no dampers.

4. The alternator is connected to an infinite bus bar.

5. The alternator flux linkages are not affected by speed changes.
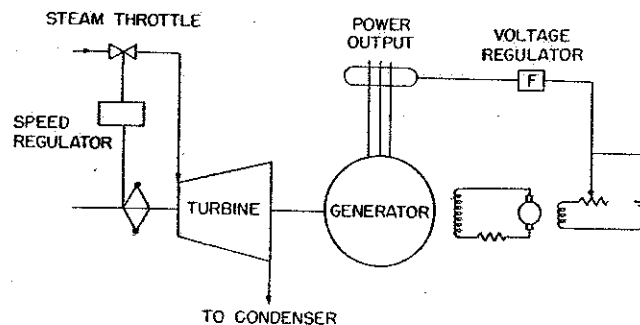
Fig. 16. Schematic of Turbogenerator
Unit.

[27] Brown, D. H., Transient Thermodynamics of Reactors and Process Apparatus, Nuclear Eng. and Sci. Conf., Preprint 81, Philadelphia (1957).
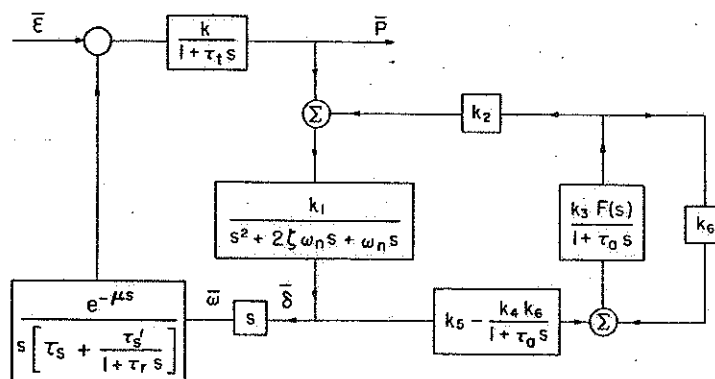
Fig. 17. Turbogenerator Transfer Functions.

The valve opening-turbogenerator power transfer function is

$$\frac{\overline{P}}{\overline{\epsilon}} = \frac{k}{1 + \tau_t s} \tag{32}$$

and is derived in reference 28.

The shaft speed-throttle opening transfer function is

$$\frac{\overline{\epsilon}}{\overline{\omega}} = \frac{e^{-\mu s}}{s \left[\tau_s + \dfrac{\tau's}{1 + \tau_r s} \right]} \tag{33}$$

and is derived in reference 29.

The power-shaft position transfer function is given in reference 30:

$$\frac{\overline{\delta}}{\overline{P}} = \frac{\dfrac{k_1}{s^2 + 2\zeta\omega_n s + \omega_n^2}}{1 - \dfrac{k_1 k_2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \left[\dfrac{k_3 F(s)}{1 + \tau_a s - k_3 k_6 F(s)} \right] \left[k_5 - \dfrac{k_4 k_6}{1 + \tau_a s} \right]} \tag{34}$$

[28] Hoyt, P. R. et al, Dynamic Field Tests of a Steam Turbine, ASME paper 57-IRD-15 (1957).

[29] Willems, P., Effect du Contrôle à Asservissement de la Turbine d'un Groupe sur la Stabilité de son Alternateur, Revue E 1, 4, 75 (1954).

[30] Heffron, W. G. and R. A. Phillips, Effect of a Modern Amplidyne Voltage Regulator on Turbogenerators, AIEE Trans., 71 III, 692 (1952).

Here, $F(s)$ = voltage comparator transfer function.

If the effects of speed changes were included (assumption e) the shaft position-power transfer function would have been much more complicated.[31]

Other authors have also treated the problem of generator dynamics under a variety of assumptions.[32,33]

Condenser Dynamics

For all practical purposes, the dynamic operation of a condenser is similar to the operation of a heat exchanger. Therefore the previous discussion on Heat Exchanger Dynamics is directly applicable to the problem of condensers.

Conclusions

Representative transfer functions of the major components of nuclear reactor plants have been described and some useful techniques for their manipulation illustrated. Both distributed and lumped parameter models have been considered.

It is shown, by simple methods, that both heat of radioactive decay and delayed neutron precursors have a stabilizing influence on reactor dynamics.

Transfer functions of pipes, mixing volumes and auxiliary equipment are not discussed, but can be found in the literature.[11, 23, 34-36]

[31] Mesarovic, M., and I. Obradovic, Influence of a Supplied Power System on the Frequency Control, C.I.G.R.E. (1956).

[32] Riaz, M., Analogue Computer Representation of Synchronous Generators, AIEE Trans. 75, III, 1178 (1956).

[33] Breedon, D. B., and R. W. Ferguson, Fundamental Equations for Synchronous Machines, AIEE Trans. 76, III, 297 (1957).

[34] Thaler, G. J. and W. A. Stein, Transfer Function and Parameter Evaluation for D. C. Servomotors, AIEE Trans. 74, II, 410 (1956).

[35] Ezekiel, F. D., and H. M. Paynter, Computer Representation of Engineering Systems Involving Fluid Transients, ASME paper 56-A-120 (1956).

[36] Chestnut, H. and R. W. Mayer, Servomechanisms and Regulating System Design, John Wiley & Sons, Inc., New York (1948).

The usefulness of distributed parameter models may be inferred from the following facts:

1. The transfer functions introduce phase shifts larger than 90° for large frequencies. This is not the case for lumped parameter models. Therefore the transcendental functions or their approximations should be used in questions of stability.

2. The transfer functions require more than one time constant for their representation. This is important when interpreting experimental data because certain characteristic frequencies may be attributed to non-existing physical phenomena.

3. The initial transient response is better represented by distributed parameter models. Disturbances do not appear to propagate instantly through the system, as is the case with a lumped parameter model.

Of course all the advantages of distributed parameter models are gained at the expense of simplicity.

Finally, it should be emphasized that each component participates effectively in the plant dynamics only over a limited frequency range. Therefore, in analyzing the plant operation, one may concentrate on definite frequency ranges and ignore effects which happen to be ineffective in that range.

Nomenclature

Section - Transfer Functions of a Distributed Parameter Reactor Core

$$b = \frac{hA^*}{M'c'}$$

c'	specific heat of coolant
h	heat transfer coefficient at the coolant-moderator interface
$k = k_1, k_2$	conductivity of moderator and fuel, respectively
ℓ	average length of coolant slabs
v, v'	coolant velocity and velocity increment, respectively
y_1, y_{12}	thickness of moderator and fuel slab, repectively

$$y_0 = \frac{\alpha}{k}\left[\frac{k_1 y_1}{\alpha_1} + \frac{k_2 y_{12}}{\alpha_2}\right]$$

A^*	effective surface of heat exchange between moderator and coolant

M_f, M_m, M' mass of fuel, moderator, and coolant, respectively

Q_0, Q steady-state and incremental heat source density in the fuel, respectively

T_{AVf}, T_{AVm} average temperature of fuel and moderator, respectively

α_1, α_2 thermal diffusivity of moderator and fuel, respectively

$$\frac{1}{\alpha} = \frac{M_m \frac{1}{\alpha_1} + M_f \frac{1}{\alpha_2}}{M_m + M_f}$$

θ_0, θ_ℓ input and output coolant temperature, respectively

ρ^*, ρ reactivity

Section - Stabilizing Effects of Heat of Radioactive Decay

τ_1 fuel time constant

τ_2 fuel-coolant interface time constant

τ_0 coolant transit time

Section - Effects of Delayed Neutrons on Stability

τ_i time constants, functions of pressure and power

Section - Heat Exchanger Dynamics

$$a_i = \frac{h_i A_i}{M_i c_i}$$

$$b_1 = \frac{h_1 A_1}{M_3 c_3}; \quad b_2 = \frac{h_2 A_2}{M_2 c_2}; \quad b_3 = \frac{h_3 A_3}{M_4 c_4}$$

c_1, c_2, c_3, c_4 specific heat of water, hot fluid, tube, and shell, respectively

h_1, h_2, h_3 heat transfer coefficient on the water side, hot fluid side, and shell side, respectively

ℓ, ℓ', ℓ'' superheater, boiler, and economizer length, respectively

v_1, v_2 water and hot fluid velocity, respectively

A_1, A_2, A_3 — surface area of tube on water side, of tube on hot fluid side, and of shell, respectively

F — fluid flow through mixing volume

M_1, M_2, M_3, M_4 — mass of water, hot fluid, tube, and shell, respectively

V — mixing volume

$\theta_{10}, \theta_{1\ell}$ — inlet and outlet water temperature, respectively

$\theta_{20}, \theta_{2\ell}$ — inlet and outlet hot fluid temperature, respectively

$$f_1 = \frac{1}{v_1}\left[s + a_1\left[(s+b_2)/(s+b_1+b_2) \right] \right]$$

$$f_2 = \frac{1}{v_2}\left[s + a_2\left[(s+b_1)/(s+b_1+b_2) \right] \right] + a_3\left[s/(s+b_3) \right]$$

$$f_3 = \frac{1}{v_1}\left[a_1 b_2/(s+b_1+b_2) \right]$$

$$f_4 = \frac{1}{v_2}\left[a_2 b_1/(s+b_1+b_2) \right]$$

$$f_5 = s + a_2\left[(s+b_1)/(s+b_1+b_2) \right] + a_3\left[s/(s+b_3) \right]$$

$$\alpha = \frac{f_2 - f_1}{2}, \quad \beta = \frac{1}{2}\left[(f_1+f_2)^2 - 4f_3 f_4 \right]^{\frac{1}{2}}$$

$$\sinh \lambda = \beta / \sqrt{f_3 f_4}$$

Discussion of Paper
Presented by Mr. Gyftopoulos

MR. ERGEN:

In the equations which you had, about in the middle of your talk, I think, you did not consider the transfer of the heat from one region, which was one time constant, into the other time constant. The heat was generated simultaneously in both regions, is that right?

MR. GYFTOPOULOS:

That is right. In this model I did not assume any time delay. However, in the previous example, which was somewhat simpler, this was taken into account. There, three time constants were considered, one for the fuel, one the fuel film coefficient, and one the transit time.

MR. SANDMEIER:

I would like to ask a question. Since all the experts in measurement are here, I wondered are there somewhere reactor data that came off in which one was allowed to insert enough reactivity that harmonics were actually generated in the flux? I am referring, for example, to the experiment which we did at EBWR. When they came into the resonance region the stroke was too large, and the harmonics were generated, producing data which I used. I wondered if such data are available somewhere, in which harmonics were actually generated; in which one was allowed to do that sort of thing?

MR. WASSERMAN:

We did some experiments in which we varied the amplitude of the input at very low power. I will talk a little bit about those this afternoon, so perhaps we will defer it until then.

MR. CORDY:

Again on the KEWB reactor, we modulated and got a lot of data. It hasn't been really looked into, but we have it.

MR. GRACE:

I wanted to ask, what is your interest in generating the transfer functions for the steam system component, the generator, and so forth?

MR. GYFTOPOULOS:

Let me see if I can understand your question. You mean my personal interest?

MR. GRACE:

What is the application of the generation?

MR. GYFTOPOULOS:

Well, I thought, since the topic of my talk was transfer functions of nuclear plant components, I might complete the issue by presenting transfer functions for all the components.

MR. GRACE:

I think that we did find in some of our early studies of power plants, some of the subsequent power plants, that the time constants associated with the components in the secondary side were rather insignificant. That the steam generator could be represented very simply with a two or three-term transfer function, which gave an attenuation of the temperature on the primary side, that is, between heat exchanger inlet temperature to heat exchanger outlet temperature, which is subsequently fed back in the reactor, and that heat exchanger transfer function, which really reflects everything on the secondary side, was quite simple.

MR. GYFTOPOULOS:

Is this experimental or analytical?

MR. GRACE:

All of this was done analytically at first, and then was checked experimentally in the plant at Idaho.

MR. GYFTOPOULOS:

There is one comment I have to make about heat exchangers. The dynamics of heat exchangers are based on heat storage and mass transfer. Therefore, if you analyze only these two phenomena, the chances are you will end up with something which behaves monotonically and introduces some reduction in the gain. However, when you proceed with models which involve also hydraulic or mechanical effects, then it has been shown that you may have coupling between the two and sustained oscillations may develop. This is a case where the model presented in the paper would be useful.

There is another reason why we have an interest in heat exchangers. This is rather from a microscopic point of view as compared to the macroscopic approach of transfer functions, and refers to the type of temperature gradients that one has at the interface between boiling and economizing regions or between boiling and superheating regions. Tube temperature gradients arise because of the change of the characteristics of the heat transfer process. The analysis shows that there is a large tube temperature gradient right at the boundary. Therefore, an oscillating transient constitutes a cyclic thermal load for the tube. We would like to have a better knowledge of this phenomenon. We are doing some work connected with the Enrico Fermi reactor heat exchanger. It is evident that such problems can be handled only with distributed parameter models.

MR. GRACE:

I think it is fortunate in those cases you can separate the stability problems on the secondary side from those on the primary side. Oscillations in heat exchanger systems have been observed due to, perhaps, poor setting of the controller, the water controller. These can be generalized, and once they are designed to be stable, their effect on the primary loop is generally quite small.

MR. GYFTOPOULOS:

You are absolutely right. I didn't read my paper, but there I concluded by saying: "It should be emphasized that each component participates effectively in the plant dynamics only over a limited frequency range. Therefore, in analyzing the plant operation, one may concentrate on definite frequency ranges and ignore effects which happen to be ineffective in that range."

MR. GRACE:

I have one more question. In your treatment of the distributed parameter effects, have you tried sectionalization, say the length of the coolant channel or the length of the boiler tube?

MR. GYFTOPOULOS:

No, because we have been able to integrate the equations for the plane geometry without having to go into this sectionalization. Actually, in a sense, it would amount to the same thing because, if you sectionalize, you might have to take three or four sections, and to the first approximation, each section introduces, say, one time lag. Now, when you have a distributed parameter system, you end up with a transcendental transfer function, and in order to have a good lumped approximation, you need a couple or three time constants. So in effect you get dispersion in characteristic frequencies, in a different way, but you end up with similar expressions.