# A Digital Nuclear Reactor Control System

E. P. GYFTOPOULOS
NONMEMBER AIEE

P. M. COBLE
NONMEMBER AIEE

THE GREAT EXPANSION in the nuclear reactor field places increased emphasis on the problem of reactor control. The complexity of a nuclear reactor installation suggests the use of digital computation in the over-all power-plant control system. The chief advantages to be gained from the use of a digital system would be a sizable increase in the flexibility of the programing of the control system and an increase in the accuracy of the computations.

The problem of digital control for nuclear reactors is approached in this paper in the following manner. First, a pressurized water-type reactor, and in particular the reactor now in operation at Shippingport, Pennsylvania, is selected as the one upon which the study is based. Certain variables which can be used to describe the behavior of such a reactor are selected as essential to the design of a reactor control system. These variables are either measured directly (e.g., neutron level, coolant temperatures) or calculated from measurable variables (e.g., reactor period, power level in the power plant). Next, several commands (e.g., fast or slow insertion of control rods, changes in flow rates) which can be used to control the behavior of the reactor are chosen. Then, an organization of digital computer elements for calculating the nonmeasurable reactor variables from the measurable ones and relating both measured and calculated quantities to the reactor control commands, are developed. A functional design of the necessary computer logic is carried out, and the required register capacities, operation frequencies, and scale factors are determined. The effect of both control system frequency and gain on the reactor stability is considered for a simplified system, and a generalized transfer function for a stability analysis of any digital reactor control system is presented.

Paper 58-1362, recommended by the AIEE Feedback Control Systems Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Winter General Meeting, New York, N. Y., February 1–6, 1959. Manuscript submitted December 1, 1958; made available for printing April 27, 1960.

E. P. GYFTOPOULOS and P. M. COBLE are with Massachusetts Institute of Technology, Cambridge, Mass.

## Quantities Chosen to Describe Reactor Behavior[1–3]

The measurable quantities include neutron density [neutrons per cm (centimeters)$^3$], coolant inlet and outlet temperatures, reactor fuel temperature, moderator temperature, control-rod temperature, shield temperature, pressures and flow rates of both primary and secondary coolants, primary coolant level, control-rod positions, and gamma-ray flux in such places as the control room or exit air duct.

The calculated quantities include the reactor period as given by:

$$\tau = \frac{1}{\dfrac{dn}{dt} \times \dfrac{1}{n}} \tag{1}$$

the reactor power output as expressed by the relation:

$$P = F(T_h - T_c) \tag{2}$$

the average coolant temperature defined as:

$$T_{avg} = \frac{T_h + T_c}{2} \tag{3}$$

and an error temperature given by the difference between the desired, or reference, average coolant temperature and the actual coolant temperature in the form:

$$\Delta T = T_{ref} - T_{avg} \tag{4}$$

Table I. Quantities Indicative of Reactor Behavior

| | Variable | Symbol | Units |
|---|---|---|---|
| Measured reactor variables | Neutron level | $n$ | neutrons per cm$^3$ |
| | Primary coolant flow rate | $F$ | gallons per minute |
| | Primary coolant inlet temperature | $Tc$ | degrees F |
| | Primary coolant outlet temperature | $Th$ | degrees F |
| | Fuel temperature | $Tf$ | degrees F |
| | Primary coolant level | $L$ | % maximum level |
| | Primary coolant pressure | $pc$ | pounds per square inch |
| | Secondary coolant pressure | $ps$ | pounds per square inch |
| | Gamma radiation flux | $\phi$ | mr* per hour |
| | Regular rod position | $X$ | cm |
| | Shim rod position | $X_S$ | cm |
| Calculated reactor variables | Reciprocal period | $1/\tau$ | seconds$^{-1}$ |
| | Average coolant temperature | $T_{avg}$ | degrees F |
| | Reactor power | $P$ | kw |
| | Proportional-plus-derivative control signal | $S_1$ | |
| | Error temperature | $\Delta T$ | degrees F |
| | Integral-plus-derivative control signal | $S_2$ | |

*Milliroentgens.

See Table I for explanation of nomenclature.

The desirable signal to govern the movement of the control rods has been found to include a component proportional to the rate of change of the neutron level as well as to the actual level itself. It is given by:

$$S_1 = n + \frac{K_1}{\tau} \tag{5}$$

where $K_1$ is a constant chosen to include a desirable fraction of the neutron level derivative component in the composite signal $S_1$.

Another type of control-rod actuating signal which is sometimes used contains a component proportional to the rate of change of the neutron level and another proportional to the error temperature.[4] It affords both derivative and integral control and is given by:

$$S_2 = \Delta T + \frac{K_2}{\tau} \tag{6}$$

## Reactor Control Commands

The commands chosen to control the reactor are as follows:

SCRAM: This is an emergency situation in which there is grave danger if the reactor is not shut down immediately. All rods are to be inserted into the reactor very fast in order to reduce the reactivity as quickly as possible.

REVERSE: The reactor is shut down, but more slowly than in a SCRAM. This situation is caused by a difficulty which requires the reactor to be shut down, but which is not as urgent as the one requiring a SCRAM.

CUTBACK: The power level in the reactor is reduced to a given low level, but the reactor is not completely shut down. The difficulty can be corrected while the reactor is still operating at very low power.

REGULATOR RODS IN: This command is caused by a power level slightly greater than

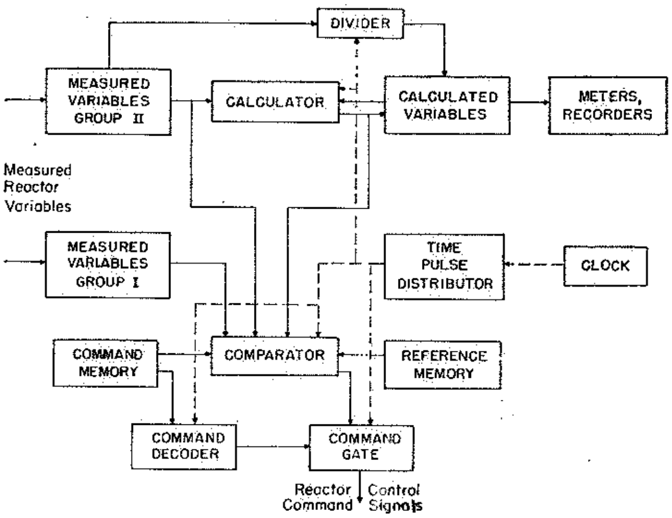


**Fig. 1. Block diagram for over-all computer**

the demand reference level. The rods are moved in an inward direction until the power level has returned to below the tolerated upper limit for the given power demand.

REGULATOR RODS OUT: Here the power level is slightly less than the given command, and the rods are moved in an outward direction until the power level has returned to above the tolerated lower limit for the given demand level.

EXCHANGE OUT: When a regulator rod has moved to its outermost position, its worth in reactivity is exchanged with that of the shim rods by moving the regulator rod from its outermost to its innermost position and at the same time moving the shim rods outward far enough to furnish the reactor with an amount of reactivity equal to that given up in the inward motion of the regulator rod. This command is necessary to accommodate the gradual outward movement of the regulator rod due to fuel depletion and poison buildup in the reactor.

EXCHANGE IN: The regulator rod is moved from its innermost position to its outermost one, while the shim rods are moved in by an amount necessary to make the net reactivity change zero.

INCREASE COOLANT FLOW RATE: The flow of the primary coolant is to be increased, for example by increasing pump speed.

DECREASE COOLANT FLOW RATE: The flow rate of the primary coolant is to be decreased.

In addition to these commands, the following subcommands are included:

INCREASE FLOW RATE (F): According to this subcommand coolant flow-rate conditions merit an increase in the flow rate, but before it is permissible to give a definite command signal to increase the coolant flow rate, certain temperature conditions must be fulfilled.

INCREASE FLOW RATE (T): Temperature conditions merit an increase in coolant flow rate, but before a definite command signal may be given, the coolant flow-rate conditions must also merit an increase in the flow rate.

The simultaneous occurrence of the subcommands INCREASE FLOW RATE (F) and INCREASE FLOW RATE (T) cause the full command INCREASE COOLANT FLOW RATE to be given. When the subcommands are not given coincidently they have no effect.

The following two subcommands are also available: DECREASE FLOW RATE (F) and DECREASE FLOW RATE (T). The simultaneous occurrence of the subcommands causes the DECREASE COOLANT FLOW RATE command to be given; their individual occurrence has no effect.

## Relations Between Reactor Variables and Control Commands

The reactor variables initiate the control commands when the variable quantities become either greater or smaller than given references. To illustrate with some specific examples, a SCRAM command is given when the neutron level is greater than 150% of full power, a REVERSE command is given when the neutron level becomes greater than 125% of full power; and a REVERSE command is also made when the coolant flow rate becomes less than 70% normal. A complete listing of the relations between the reactor variables and the control commands is given in Appendix I, which gives the conditions on the magnitudes of the variables which initiate the various commands. In the nomenclature used, the symbols designating reactor variables are those given in Table I, and the additional subscripts are defined as follows: $m$ refers to the absolute maximum, or SCRAM level; $r$ the REVERSE level; $c$ the CUTBACK level; $h$ the higher of two variable levels which cause the same command; $o$ refers to the level which initiates the EXCHANGE OUT command; $i$ the level which initiates the EXCHANGE IN command, and "ref" to the reference power-demand signal from the turbogenerator. Numbered subscripts refer to various trip levels used in initiating the subcommands; while $\delta$ is a small fraction used in establishing a zone, or region, of permitted variable values. For example, $p_{crl}$ refers to the lower REVERSE level for the coolant pressure variable.

## General Layout of Control Computer

### General Description

A detailed block diagram of the over-all computer is shown in Fig. 1. The solid lines indicate the flow of processed information, while the dotted lines show the flow of the timing pulses necessary to operate the computer. The input to the computer, which consists of the measured reactor variables given in Table I, is sent directly to measured variable storage. The variables are categorized into two groups. Group I includes those variables which are not used in computing the calculated variables, whereas group II consists of those variables which are used in the calculations. The variables in group II are the neutron level, the primary coolant flow rate, the reactor inlet coolant temperature, and the reactor outlet coolant temperature. The remaining variables are included in group I.

The calculations are performed in two computing elements: a calculator which performs addition, subtraction, and multiplication; and a divider which performs division. The need for the two separate computing elements arises from the fact that the necessary division in the computation of the reactor period takes longer than the total time required to perform all other calculations. Also, the calculator, which is presently quite simple, would have to be substantially enlarged if it were to perform division in addition to the other arithmetic operations. The calculator and divider feed the calculated variable storage registers, the signals from which are sent to various indicating and recording devices.

The reactor variables (both measured and calculated) are sent to the comparator where they are compared with the reference levels. The comparisons are carried on at the same time the calculations are being performed in the calculator and the divider. Each reference level has an associated coded command signal, which is read partly into the comparator and partly into the command decoder every time a comparison is made. The result of the comparison is gated with the decoded command signal to provide computer output signals which either initiate or terminate the reactor control com-

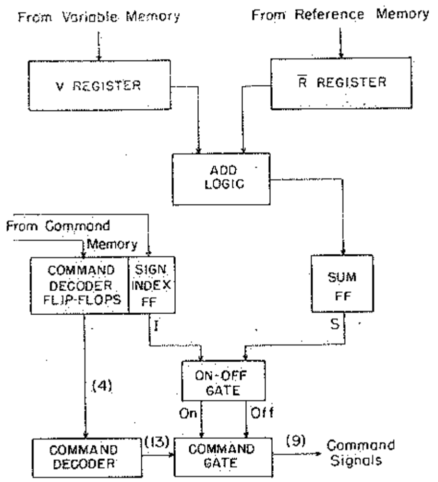


**Fig. 2. Comparator and its associated elements**

mand signals. Timing pulses for all operations are furnished by a 64-kc clock and its associated time-pulse distributor. All operations are carried out in parallel form.

### Calculator

The operations of addition, subtraction, and multiplication are performed in the calculator, which functions as an accumulator and which consists of a sum-product register, an addend-multiplicand register, and a multiplier register. In the process of adding two numbers, one number is read into the sum register while the other number is placed in the addend register. The logical expressions for the sum and carry for each digit position are formed when the numbers are inserted in their respective registers, and the actual addition operation is performed upon the application of an "add" pulse. The sum is stored in the sum-product register and may be read out upon the application of a "read-out" pulse. Subtraction is performed by complement addition. Multiplication consists of a series of additions of the contents of the multiplicand register to those of the product register, and shifts of the product and multiplier register digits according to whether the multiplier digits are ones or zeros.

### Divider

Division is performed in the divider by a series of additions and subtractions governed by the following rules.[5] First, the divisor is subtracted from the dividend. If the remainder is positive, a one is placed in the most significant quotient position and the next operation is set up as a subtraction. If the remainder is negative, a zero is placed in the quotient and the next operation is set up as an addition. The divisor is then shifted and the process repeated, with the "answer" digit being placed in the next most significant quotient position.

### Comparator and Associated Elements

The function of the comparator is to compare the reactor variables with various reference levels, and to initiate control command signals when the variables exceed (or in some cases become less than) the reference levels. The comparison is made by subtracting the reference level from the variable level. In the actual computation the complement of the reference level is added to the true value of the variable level.

As is shown in Fig. 2, the variable is placed in the $V$ register and complement of the reference level in the $\bar{R}$ register. The addition is performed by means of add logic, which sends the sign of the remainder to the sum flip-flop. Each control command is given a specific code. If the control command signal is to be given when the variable exceeds the reference level, a sign index of 0 is assigned to the control command. If the signal is to be turned on when the variable becomes less than a particular reference value, a sign index of 1 is assigned. A list of all possible commands, their binary code, and sign index number is given in Table II.

These command codes and sign indexes are stored in the command memory. When a particular comparison is to be made, the command code and sign index associated with the reference level used in the comparison are read into the command decoder and sign index flip-flops. The command signal is turned on if the digits in the sign index and sum flip-flops are both 1 or both 0. The command signal is turned off if one flip-flop contains a 1 and the other a 0. This gating action is performed by the on-off gate. The command decoder is a diode matrix which causes one of its 13 output lines to be activated in response to inputs from the four command decoder flip-flops. The command gate functions to turn the command signals on and off at the proper times according to signals from the on-off gate and from the command decoder.

## Timing-Pulse Generation

The timing pulses necessary for operation of the computer are furnished by a clock and its associated time-pulse distributor, the latter consisting of a six-stage flip-flop binary counter and an associated diode matrix. Output pulses, designed $P1$ through $P64$, are furnished and are routed to different places in the computer. Since the clock frequency is 64 kc, a complete cycle of timing pulses lasts 1 ms (millisecond).

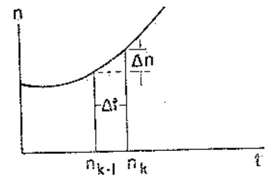


**Fig. 3. Sampling for period calculation**

## Calculation of Reactor Variables

### Reactor Period

In the determination of the reactor period, due to ease of computation, the reciprocal period $1/\tau$ rather than the period itself is calculated. The digital computation is performed according to the approximate relation:

$$\frac{1}{\tau} \approx \frac{\Delta n}{n\Delta t} \approx \frac{n_k - n_{k-1}}{n_{k-1}} \times \frac{1}{\Delta t} \tag{7}$$

A hypothetical waveform of neutron level as a function of time is shown in Fig. 3. Samples of the neutron level are made every millisecond. The $(k-1)$th sample is subtracted from the $k$th sample, the difference is divided by the $(k\text{-}1)$th sample. The $(k-1)$th sample, rather than some mean value between the $(k-1)$th and $k$th is used in the denominator in order to ensure that any errors in period for an increasing neutron level will always be on the "safe" side. In other words, for an increasing neutron level, $\Delta n$ will always be divided by the smaller of the two neutron levels, thus making $1/\tau$ greater than it would be if $n_k$ or some mean value of $n$ were used in the denominator.

The actual sequence of operations for performing the calculation of reactor period is as follows. On timing pulse $P64$ the neutron level is sampled, and a number proportional to the neutron level is read into the $n_k$ storage register com-

**Table II. Command Codes and Sign Indexes**

| Command | Code | Sign Index |
|---|---|---|
| INCREASE F | 0001 | 1 |
| DECREASE F(F) | 0010 | 0 |
| INCREASE F(F) | 0011 | 1 |
| REVERSE (variable reference) | 0100 | 0 |
| | 0100 | 1 |
| INCREASE F(T) | 0101 | 0 |
| DECREASE F(T) | 0110 | 1 |
| RODS IN | 0111 | 0 |
| RODS OUT | 1000 | 1 |
| DECREASE F | 1001 | 0 |
| SCRAM | 1010 | 0 |
| EXCHANGE IN | 1011 | 1 |
| EXCHANGE OUT | 1100 | 0 |
| CUTBACK (variable reference) | 1101 | 0 |
| | 1101 | 1 |

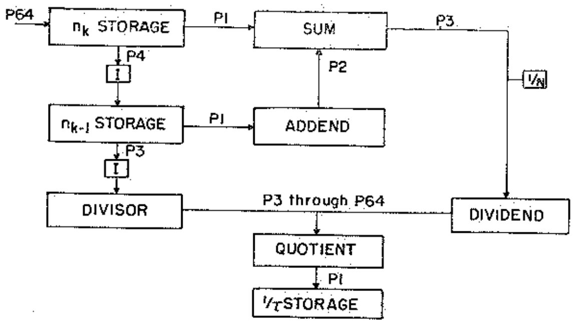


**Fig. 4 (left). Sequence for period calculation**

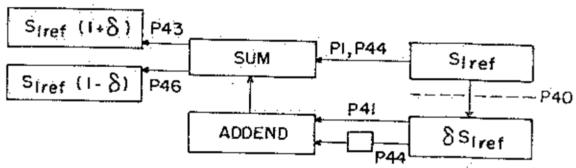


**Fig. 5 (right). Sequence for reference control-signal calculation**

posed of a parallel array of flip-flops (see Fig. 4). This operation may take the form of sending the output signal from an ionization chamber through an analog-to-digital converter. The analog-to-digital conversion problem is not discussed here but would have to be considered before a physical system could be realized.

On the following timing pulse, $P1$, the present neutron level $n_k$ is read into the sum register of the calculator and the complement of the previously sampled neutron level, $\bar{n}_{k-1}$, is read out of the $\bar{n}_{k-1}$ storage register and into the calculator addend register. $P2$ is an add pulse which causes the contents of the addend register to be added to the contents of the sum register, the result being stored in the sum register. In this case the result is $n_k - n_{k-1}$. This quantity is read into the dividend register in the divider on $P3$. If $n_k - n_{k-1}$ is positive, $P3$ also reads a 0 into the sign position of the quotient register since $n_{k-1}$ is always positive; hence the sign of the quotient depends only on the sign of $n_k - n_{k-1}$. If $n_k - n_{k-1}$ is negative, its complement is stored in the sum register and must be inverted before being read into the dividend register. The $I/N$ block in Fig. 4 means the digits are to be inverted (complemented) as they are read into the dividend register if the sign is negative. $P3$ also reads $n_{k-1}$, the complement of $\bar{n}_{k-1}$, into the divisor register.

$P4$ reads the complement of $n_k$ into the $\bar{n}_{k-1}$ register where the present $n_k$ becomes the $n_{k-1}$ for the next computation. The $n_k$ register receives the next $n_k$ sample on $P64$. During pulses $P3$ through $P64$ the division which forms

$$\frac{n_k - n_{k-1}}{n_{k-1}}$$

is carried out, and on the following $P1$ the quotient is read into the $1/\tau$ flip-flop storage register.

### Additional Computations Performed by Calculator

The additional reactor control variables which must be calculated are the average coolant temperature, the coolant temperature difference between the inlet and outlet to the reactor, the reactor power, and the proportional-plus-derivative control-rod actuating signal.

The calculation of the proportional-plus-derivative control signal proceeds as follows. On $P4$ the present neutron level is read out of the $\bar{n}_k$ register into the sum register. At the same time the reciprocal period is read from the $1/\tau$ storage register to the addend register. Information is stored in the $1/\tau$ register in the form of the sign digit followed by the magnitude of the number in ordinary binary code (not complemented if negative). Therefore if the reciprocal period is negative, the digits must be complemented before being read into the addend register. $P5$ adds the two quantities, after which $P6$ reads the sum $n + K_1/\tau = S_1$ from the sum register to the $S_1$ register. In the read-out operation the digits are complemented if the sign is negative.

In the computation of the average coolant temperature, $T_h$ is read into the sum register and $T_c$ is sent to the addend register on $P7$. $P8$ adds the two quantities and stores twice the average temperature in the sum register. $P9$ reads out the average temperature to the $T_{avg}$ storage register, the factor of 2 being eliminated by ignoring the least significant bit in the read-out process.

The calculation of the coolant temperature difference involves reading in $T_h$ to the sum register and the complement of $T_c$ to the addend register on $P10$, adding these quantities on $P11$, and reading $(T_h - T_c)$ out of the sum register on $P12$. The computation of reactor power is slightly more involved. $P12$ also reads $(T_h - T_c)$ from the sum register to the multiplier register and puts the coolant flow rate $F$ into the multiplicand register. The following odd-numbered pulses are "add attempts," i.e., the least significant digit in the multiplier register is tested; if this digit is a 1, an "add" pulse is applied to the calculator, while if a 0, no pulses occur. The even-numbered pulses shift the contents of the multiplier and product registers. The multiplication is completed on $P39$; hence $P40$ reads the reactor power out of the sum-product register.

The calculator must perform two additional computations, namely the calculation of the tolerated upper and lower limits for the reference proportional-plus-derivative control signal, which varies according to the load on the turbogenerator. The sequence of computations is illustrated in Fig. 5 and proceeds as follows. On $P40$ the signal $S_{1ref}$ is read into the $\delta S_{1ref}$ register. For the present computations $\delta$ has been chosen as 1/16. The multiplication of $S_{1ref}$ by $\delta$ is accomplished by dividing by $1/\delta$. Since $1/\delta = 16$, all that is required for this operation is to ignore the four least significant bits in the transfer of information from the $S_{1ref}$ register to the $\delta S_{1ref}$ register. The $\delta S_{1ref}$ register is made four bits shorter than the $S_{1ref}$ register and the transfer process in effect shifts $S_{1ref}$ to the right by four bits. $P41$ reads $S_{1ref}$ into the sum register and $\delta S_{1ref}$ into the addend register. $P42$ performs the addition, and $P43$ reads $S_{1ref}$ $(1+\delta)$ out of the sum register. $P44$ reads the complement of $\delta S_{1ref}$ to the addend register and $S_{1ref}$ to the sum register. After the addition is performed on $P45$, the result $S_{1ref}$ $(1-\delta)$ is read out of the sum register on $P46$. A complete pulse-timing sequence for the comparator operation is given in Appendix II.

## Timing Sequence for Comparator

The principles of comparator operation have already been discussed. The actual

**Table III. Comparator Command Timing Sequence**

| Command | Pulses on Which Command Can Occur |
|---|---|
| INCREASE F | P41 |
| DECREASE F(F) | P42 |
| INCREASE F(P) | P43 |
| REVERSE (variable > reference) | P2, 5, 8, 12, 14, 16, 18, 22, 26, 28, 29, 33 |
| REVERSE (variable < reference) | P20, 24, 35, 39, 45 |
| INCREASE F(T) | P31 |
| DECREASE F(T) | P32 |
| RODS IN | P10 |
| RODS OUT | P11 |
| DECREASE F | P44 |
| SCRAM | P1, 4, 7 |
| RODS EXCHANGE IN | P38 |
| RODS EXCHANGE OUT | P37 |
| CUTBACK (variable > reference) | P3, 6, 9, 13, 15, 17, 19, 23, 27, 30, 34 |
| CUTBACK (variable < reference) | P2, 1, 25, 36, 40, 46 |

timing sequence for comparator operation is now given. Appendix III shows what reference levels are read into the $\bar{R}$ register on what timing pulses. Table III lists the timing pulses which read the various command codes into the command decoder. By comparing Appendixes II and III it may be seen that the quantities which can cause scramming of the reactor, namely excess neutron level or reactor period, are tested as soon as possible after the sampling of $n$ and the completion of the computation of $1/\tau$. Also, note that the control signal $S_1$ is computed on $P6$ and sent to the comparator on pulses $P7$ through $P11$.

Fig. 6 shows a block diagram with the variable storage registers and calculating registers as well as the timing pulses associated with each register.

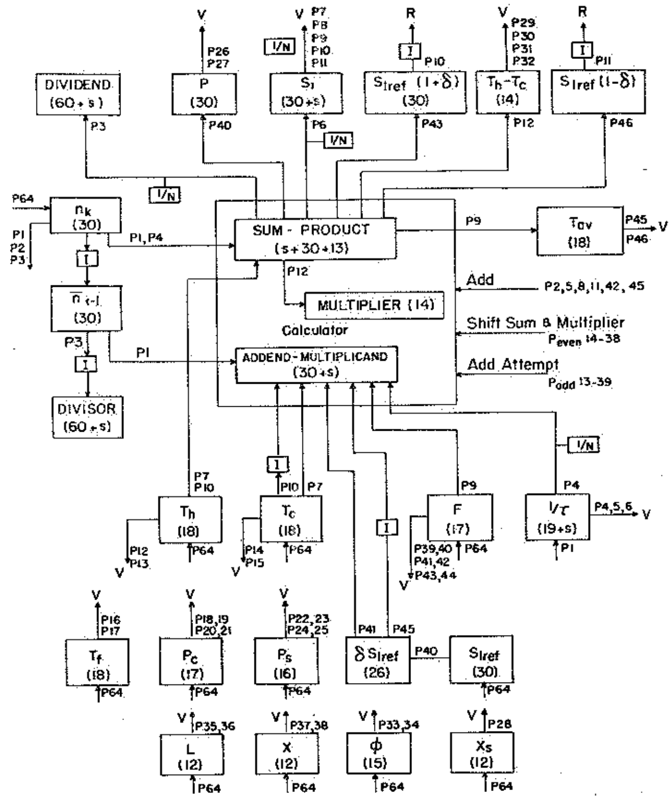


**Fig. 6. Interrelations of storage and calculating registers**

## Magnitudes of Control Quantities

Table IV gives the magnitudes of each reference level and shows the criteria used in their determination. Much of the data for these criteria applies to the Shippingport reactor.[4]

In determining the magnitude of the reactor reciprocal period, the following procedure was used. Since the variables are sampled every millisecond, the reciprocal period is:

$$(1/\tau)=\frac{\Delta n}{n}\times\frac{1}{\Delta t}=\frac{\Delta n}{n}\times\frac{1}{10^3} \tag{8}$$

or

$$\frac{\Delta n}{n}=10^{-3}\times\frac{1}{\tau} \tag{9}$$

For a reciprocal period of 0.2 second$^{-1}$, the fractional reference level must be $10^{-3}\times 0.2=0.0002$, since $\Delta n/n$ is what is actually calculated. In the calculation of the proportional-plus-derivative control signal $S_1=n+K_1/\tau$, $K_1$ is chosen so that a 22-second period is equally weighted against a full-power level. Using this criterion, $K_1$ turns out to be $2^{41}=2^{30}\times 2^{11}$. Since the reciprocal period is a fraction (and has its decimal point at the left of the 30-bit register) while the neutron level is a whole number (having its decimal point at its right), the proper scaling is accomplished by merely shifting the $1/\tau$ number 11 bits to the left as it is read into the calculator for addition to the neutron level. The capacity of each variable storage register is shown by the numbers in parentheses in Fig. 6.

## Derivative-Plus-Integral Control

As was previously mentioned, instead of employing a proportional-plus-deriva-

**Table IV. Magnitude of Reference Levels**

| Reference Level | Criteria[1–4] | Magnitude of Level | Actual No. on Computer |
|---|---|---|---|
| $n_m$ | 150% full power | $1.5\times10^8$ | $1.5\times10^8$ |
| $n_r$ | 125% full power | $1.25\times10^8$ | $1.25\times10^8$ |
| $n_c$ | 112.5% full power | $1.125\times10^8$ | $1.125\times10^8$ |
| $(1/\tau)_m$ | $\tau=5$ seconds | 0.0002 second$^{-1}$ | 0.0002 |
| $(1/\tau)_r$ | $\tau=10$ seconds | 0.0001 second$^{-1}$ | 0.0001 |
| $(1/\tau)_c$ | $\tau=15$ seconds | 0.000067 second | 0.000067 |
| $S_{1m}$ | 150% full power | $1.5\times10^8$ | $1.5\times10^8$ |
| $S_{1r}$ | 125% full power | $1.25\times10^8$ | $1.25\times10^8$ |
| $S_{1c}$ | 112.5% full power | $1.125\times10^8$ | $1.125\times10^8$ |
| $T_{hr}$ | $T_h$ maximum = 636 F | 625 F | 62,500 |
| $T_{hc}$ | | 600 F | 60,000 |
| $T_{fr}$ | $T_f$ maximum = 636 F | 625 F | 62,500 |
| $T_{fc}$ | | 600 F | 60,000 |
| $T_{cr}$ | $(T_h-T_c)$ at full power = 35 F; | 590 F | 59,000 |
| $T_{cc}$ | $T_{hr}=625$ F; $T_{hc}=600$ F | 565 F | 56,500 |
| $(T_h-T_c)_r$ | 143% full power | 50 F | 5,000 |
| $(T_h-T_c)_c$ | 126% full power | 43 F | 4,300 |
| $(T_h-T_c)_1$ | 108.5% full power | 38 F | 3,800 |
| $(T_h-T_c)_2$ | 50% full power | 17.5 F | 1,750 |
| $T_{avgr}$ | $k=0.01$ | 475 F | 47,500 |
| $T_{avgc}$ | $k=0.005$ | 500 F | 50,000 |
| $F_1$ | Flow Rate 115% Normal | 51,750 gallons per minute | 25,875 |
| $F_2$ | Flow Rate 105% Normal | 47,250 gallons per minute | 23,625 |
| $F_3$ | Flow Rate 95% Normal | 42,700 gallons per minute | 21,350 |
| $F_4$ | Flow Rate 90% Normal | 40,500 gallons per minute | 20,250 |
| $F_c$ | Flow Rate 80% Normal | 36,000 gallons per minute | 18,000 |
| $F_r$ | Flow Rate 70% Normal | 31,500 gallons per minute | 15,750 |
| $p_{crh}$ | $p_c$ maximum = 2,500 psi* | 2,400 psi | 24,000 |
| $p_{cch}$ | $p_c$ normal = 2,000 psi | 2,200 psi | 22,000 |
| $p_{ccl}$ | $p_c$ varies 1,850–2,150 psi | 1,800 psi | 18,000 |
| $p_{crl}$ | | 1,600 psi | 16,000 |
| $p_{srh}$ | $p_s$ maximum = 975 psi | 950 psi | 9,500 |
| $p_{sch}$ | $p_s$ full power = 600 psi | 925 psi | 9,250 |
| $p_{scl}$ | $p_s$ no load = 885 psi | 525 psi | 5,250 |
| $p_{srl}$ | | 450 psi | 4,500 |
| $L_c$ | | 80% | 800 |
| $L_r$ | | 70% | 700 |
| $\phi_r$ | | 5 mr†/hour | 5,000 |
| $\phi_c$ | | 2 mr/hour | 2,000 |
| $P_r$ | 150% full power | 90,000 kw | 118,125,000 |
| $P_c$ | 125% full power | 75,000 kw | 98,437,500 |
| $X_t$ | 1/4 full rod travel | 46 cm | 920 |
| $X_o$, $X_r$ | 3/4 full rod travel | 138 cm | 2,760 |

* Pounds per square inch.

† Milliroentgens.

tive signal to govern control-rod movement, a derivative-plus-integral signal may be used. In fact, such a signal is actually used in controlling the Shippingport reactor, and is given by:[4]

$$S_2 = \Delta T + K_2/\tau \qquad (10)$$

In using this type of control, it is necessary to compute the intermediate variable $\Delta T$. This can be done by reading in $T_{ref} = 525$ F (degrees Fahrenheit) and the complement of $T_{avg}$ to the sum and addend registers of the calculator and then applying the usual add pulse. $K_2/\tau$ could then be added to $\Delta T$ in the same way that $K_1/\tau$ is added to $n$.

$K_2$ may be chosen as follows. If, in the expression for $S_2$, the reactor period is replaced by the quantity $l/\delta k$, where $l$ is the mean neutron lifetime in the reactor and $\delta k$ is a small change in reactivity, the control signal becomes:

$$S_2 = \Delta T + K_2\left(\frac{\delta k}{l}\right) \qquad (11)$$

By assuming that the average coolant temperature $T_{avg}$ was initially at $T_{ref}$ but has changed by a slight amount $\Delta T'$, the change can be expressed as:

$$\Delta T' = T_{avg} - T_{ref} = -\Delta T \qquad (12)$$

The change in reactivity is related to the temperature change by

$$\delta k = (-\alpha)\Delta T' \qquad (13)$$

where $(-\alpha)$ is the temperature coefficient of reactivity of the reactor, which in the case of the Shippingport reactor is inherently negative. Equation 11 can now be rewritten as:

$$S_2 = \Delta T + K_2 \frac{\alpha}{l}\, \Delta T \qquad (14)$$

For both terms in equation 14 to carry equal weight $K_2\, \alpha/l$ must equal 1. For a temperature coefficient of $-2\times10^{-4}$ per $F$ and a mean neutron lifetime of $10^{-3}$ seconds, $K_2$ must equal 5.

## Breaking into Calculator Timing Sequence

In the design of a computer similar to the one just described, it may be desired to use the calculator to compute a very slowly varying quantity. and there will not be enough timing pulses available to program the calculation into the regular timing sequence. One way to perform the calculation and still meet the timing-pulse restriction is occasionally to break into the calculator timing sequence at a point where other slowly varying quantities are being computed and to perform the new calculation instead of the regular one. A simple method for doing this has been devised and will be discussed. A specific example will be used to illustrate the procedure.

Suppose that timing pulses $P40$ through $P46$ are not available for calculating $S_{1ref}$ $(1\pm\delta)$ from the power demand signal $S_{1ref}$, and that the power demand changes slowly enough so that it is necessary to compute $S_{1ref}$ $(1\pm\delta)$ only occasionally. Since it is not necessary to perform the regular temperature calculations on every cycle, the $S_{1ref}$ $(1\pm\delta)$ calculations will occasionally be substituted for the temperature calculations in the regular program. More specifically, it is desired occasionally to substitute the program given in Appendix IV for that of pulses $P6$ through $P12$ of Appendix II.

The logic designed to carry out this substitution functions as follows. When a "change sequence" pulse occurs, a flip-flop is set to control certain AND gates which block timing pulses $P7$, $P9$, $P10$, and $P12$ and other gates which permit pulses $P6'$, $P9'$, and $P12'$ to occur. Pulses $P6$, $P8$, and $P11$ are still allowed, and the timing sequence given in Appendix IV results. $P12'$ returns the flip-flop to its original state.

## Decreasing Permissible Reactor Periods During Low-Power Operation

The proportional-plus-derivative control signal $S_1$ is given by equation 10 as equal to $n + K_1/\tau$. $K_1$ is chosen so that a 22-second period is equally weighted against a full (100%) power neutron level. The control signal is obtained, however, according to the load on the turbogenerator. Therefore, for a power demand of less than 100% the signal $S_1$ will be decreased from its full-power value. When this happens, the tolerated limit on the reciprocal period will be automatically reduced so that the shortest allowable period will be greater than 22 seconds. For example, for a power demand of 50%, the period limit will be 44 seconds. This means that for small power demands the permissible rate of change in reactivity is decreased; hence it will take longer for the reactor to reach desired power levels.

A method has been devised for speeding up changes in power level during low-power demand. Essentially, it consists of merely modifying the calculated reactor period. Table V shows the reciprocal period demands which correspond to several given power demands in the sense of having the same controlling effect on the regulator rods, and by what fraction

**Table V. Power-Demand Reciprocal-Period Correspondence**

| Power Demand, % | Corresponding $1/\tau$ Demand | Multiply True $1/\tau$ by |
|---|---|---|
| 100 | 0.04550 | 1.0 |
| 50 | 0.02275 | 0.5 |
| 10 | 0.00455 | 0.1 |

the true reciprocal periods must be multiplied in order to tolerate a 22-second period for each power demand.

As a specific example, assume the power demand is 50% and the reactor is currently operating on a 22-second period. The calculated $1/\tau$ will be 0.0455, but if the computer described in this article is used, the control signal $S_1$ will tolerate a reciprocal period no greater than 0.02275. Hence, the control rods will be moved in until the reactor is forced to operate at nothing less than a 44-second period. Now, suppose that the calculated $1/\tau$ is multiplied by one half, the ratio of actual to 100% power demand, to obtain a new reciprocal period $(1/\tau)' = (0.0455\times{}^1/_2) = 0.02275$. When this quantity is compared with the control signal the limit is just reached, and the reactor is still allowed to operate on a 22-second period.

Using this method, the calculations would proceed as follows. The computation of $1/\tau$ would be carried out in exactly the same manner as before, the result being sent to the meters, recorders, and the $V$ register of the comparator the same as before. However, an additional quantity $(1/\tau)' = (1/\tau)$ times the power demand divided by the full-power level would be computed, and this quantity would be employed in computing the control signal $S_1$. In other words, equation 10 would be replaced by the relation

$$S_1 = n + K_1\left(\frac{1}{\tau}\right)' \qquad (15)$$

This ensures that the allowable reciprocal period is not reduced when the power demand is below that of full power.

## Stability

An essential requirement for any control system is that it be stable for all conditions for which the system is to be operated. In a nuclear reactor control system the stability requirement is even more important since the damage which could result from an unstable atomic pile is considerable.

A complete stability analysis cannot possibly be presented here; however, a brief and simplified analysis which indicates that the designed digital reactor control system is stable and which throws some light on the general problem of stability of sampled-data reactor control systems follows. In this analysis, a general block diagram for a nuclear reactor and its associated sampled-data control system is set up and reduced. The transfer functions which pertain to the specific system under consideration are formulated, and several simplifications are made. The stability is studied by means of root-locus techniques, two root loci plots being included to illustrate the effect of varying the sampling frequency on the allowable over-all system gain.

A generalized block diagram for a sampled-data nuclear reactor control system is given in Fig. 7(A). $F(s)$ is the transfer function for the nuclear reactor and relates changes in reactivity $\delta k$ to changes in neutron level $\delta n$. The output $\delta n$ is sampled by an impulse modulator $IM$ and the samples (impulses) are sent through a digital computer $G(z)$. The notation $G(z)$ is used to indicate a purely discrete (sampled-data) system, whose Laplace transform consists of functions of the form $\epsilon^{sT} = z$, where $T$ is the time between samples. The output of the computer is sent through some continuous elements, represented by $H(s)$, in the feedback loop before being used to furnish the reactor with reactivity of opposite polarity to the original disturbance.

The block diagram of Fig. 7(A) can be reduced to the successive forms shown by Figs. 7(B)–(E). The starred quantity denotes that only the samples $(HF)^*$ of the purely continuous transfer function $HF$ are of interest. The transmission $T$ from $\delta k$ to $\delta n$ may be written by inspection of Fig. 7(E) as:

$$T = F - \frac{F}{H}(MG)\left[\frac{1}{1+(HF)^*G}\right](HF) \qquad (16)$$

If the input $\delta k$ consists of one sample, a unit impulse, the transmission can be thought of as representing the unit impulse response of the system. If one is interested only in the samples of the impulse response, the pure $z$-domain transmission may be given as:

$$T^* = F^* - \left(\frac{F}{H}\right)^*\left[\frac{G(HF)^*}{1+(HF)^*G}\right] \qquad (17)$$

Hence the system is represented by an entirely discrete transfer function and root locus or similar frequency domain techniques may be employed. The next step is to determine the specific functions $F$, $G$, $H$, $F^*$, $\left(\frac{F}{H}\right)^*$, and $(HF)^*$.

The transfer function for a nuclear reactor can be shown to be of the following form:[1]

$$\frac{\delta n(s)}{\delta k(s)} = \frac{n_o}{l}\,\frac{1}{s\left[1+\sum_{i=1}^{6}\frac{\beta_i}{l(s+\lambda_i)}\right]} \qquad (18)$$

where $n_o$ is the steady-state neutron level, $l$ is the mean neutron lifetime, $\beta_i$ is the fraction of total neutrons in the $i$th group of delayed neutrons, and $\lambda_i$ is the decay constant for the $i$th group. Equation 18 can be simplified by considering one group of delayed neutrons with appropriate constant $\beta$, $\lambda$. Equation 18 becomes:

$$\frac{\delta n(s)}{\delta k(s)} = \frac{n_o(s+\lambda)}{ls(s+d)} \qquad (19)$$

In the following analysis $\lambda$ will be chosen as 0.075 second$^{-1}$, while the value of $d$ will be taken to be 50 seconds$^{-1}$. Temperature effects may also be taken into consideration, although for the purposes of this analysis these effects will be neglected.

The feedback control consisting of $G(z)$ and $H(s)$ in Fig. 7(A) actually is made up of several components. First, the computer sends out a sampled signal proportional to the neutron level and the rate of change of the neutron level. The computer transfer function will be denoted by $D(z)$.

In order to facilitate the analysis, the derivative component of $D(z)$ will be ignored and the computer will be assumed to furnish merely a delay of some sample time. In other words,

$$D(z) = \frac{1}{z} \qquad (20)$$

Next, the signals are considered to be quantized, i.e., for any neutron level greater than $n_o(1+\delta)$ the computer sends out a $+1$ signal meaning move the rods in, while for any neutron level less than $n_o(1-\delta)$ a $-1$ signal is generated to move the rods out. For neutron levels between $n_o(1+\delta)$ and $n_o(1-\delta)$ no corrective signal at all is initiated. For the purposes of this analysis, however, the quantizer will be assumed to have a transfer function of unity.

After being quantized, the control signal is held, or clamped, at a given value for an entire cycle of computer operation (sampling period). The transfer function for the clamping operation is given by $1/s\,[1-(1/z)]$.

Finally, the clamped signal is used to control a motor, which in turn drives the control rods. The transfer function used to describe the behavior of the motor is $(K_m/1+\tau_m s)$ where $K_m$ and $\tau_m$ are the gain and time constants for the motor respectively. A value of $\tau_m = 0.1$ will be used in the ensuing analysis.

The motor transfer function and the $1/s$ term from the clamper will be grouped together and termed $H(s)$, while the computer transfer function $D(z)$ will be paired with the $[1-(1/z)]$ term from the clamper to obtain $G(z)$. The quantities $F$, $G$, and $H$ can thus be expressed as:

$$F(s) = \frac{n_o(s+\lambda)}{ls(s+d)} = \frac{K_1(s+0.075)}{s(s+50)} \qquad (21)$$

$$G(z) = \frac{1}{z}\left(1-\frac{1}{z}\right) = \frac{z-1}{z^2} \qquad (22)$$

$$H(s) = \frac{K_m}{s(\tau_m s+1)} = \frac{K_m/\tau_m}{s(s+10)} \qquad (23)$$

Inspection of equation 17 reveals that the stability problem can be analyzed by focusing attention on the poles of $F^*$ and $\left(\frac{F}{H}\right)^*$ and the zeros of $1 + (HF)^*G$. $F(s)$ can be expanded in a partial fraction expansion of the form:

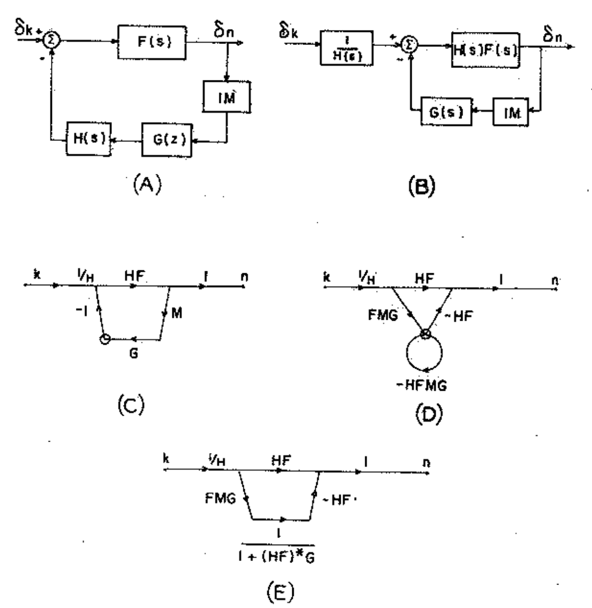


**Fig. 7. Reduction of generalized reactor-control block diagram**

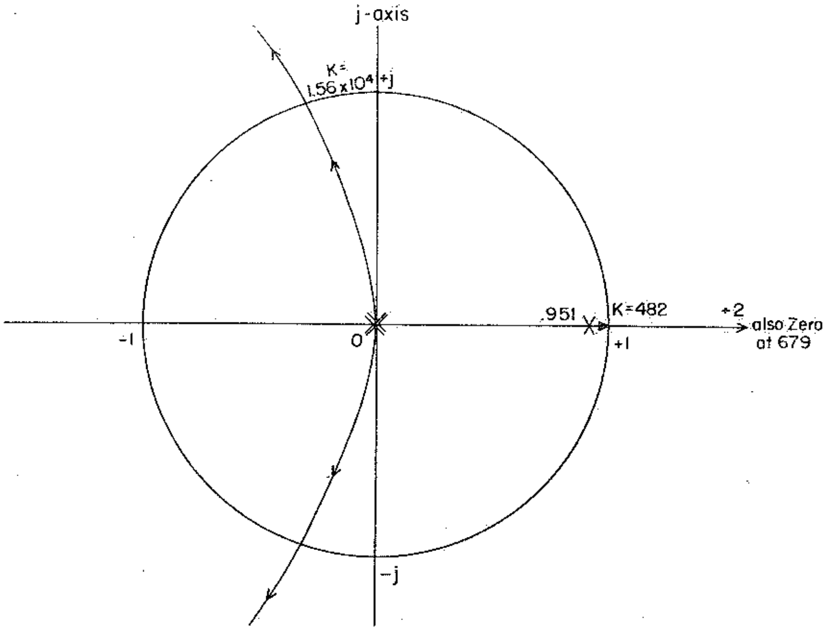


Fig. 8. Locus of roots of $1+(HF)^*G$ for $T=10^{-3}$ seconds

$$F(s)=\frac{a}{s}+\frac{b}{s+50} \tag{24}$$

The $z$-transform for $F(s)$ can then be written down by inspection as:

$$F^*(z)=\frac{a}{z-1}+\frac{b}{z-\epsilon^{-50T}} \tag{25}$$

where $T$ is the sampling period.

$F^*(z)$ is seen to have poles at $z=1$ and at $z=\epsilon^{-50T}$. For the transform $z=\epsilon^{sT}$ the unit circle corresponds to the $j$-axis of the $s$-plane, poles outside of the unit circle causing instability and poles inside of the unit circle giving rise to stable systems. Since the poles of $F^*(z)$ do not lie outside of the unit circle, $F^*(z)$ is seen to be stable. A similar argument holds for $\left(\frac{F}{H}\right)^*$.

In examining the zeros of $1+(HF)^*G$ root-locus techniques will be used, since $(HF)^*$ is a function of the feedback loop gain. $(HF)^*G$ can be shown to be given by:

$$(HF)^*G=\frac{1.5Tz}{(z-1)^2}+\frac{20}{z-1}+\frac{5}{z-\epsilon^{-50T}}$$
$$\frac{25}{z-\epsilon^{-10T}}\;\frac{(z-1)}{z^2}\times10^{-4}K \tag{26}$$

where $K$ is defined as $K_1K_m/\tau_m$. The computer was designed to sample the neutron level and send out a command signal to the control rods every millisecond. Hence $T$ is set equal to $10^{-3}$ in equation 26 and in order to simplify the analysis, $\epsilon^{-10T}=\epsilon^{-0.01}$ is approximated by unity. Equation 26 then reduces to:

$$(HF)^*G=\frac{(z-679)\times1.5\times17^{-7}K}{z^2(z-0.951)} \tag{27}$$

A plot of the locus of the roots of $1+(HF)^*G$ as given by equation 27 is shown in Fig. 8. The system is seen to be stable for all values of positive $K$ less than 482.

In order to determine the effect of varying the sampling frequency, a sampling period of $10^{-5}$ seconds was hypothesized. A frequency greater than 1 kc was chosen, rather than one smaller, since for high sampling frequencies approximations can be made which greatly simplify the analysis, whereas for sampling frequencies less than 1 kc, the analysis becomes far more complicated. For a sampling period of $10^{-5}$ seconds both $\epsilon^{-10T}$ and $\epsilon^{-50T}$ can be approximated by unity, and equation 26 reduces to:

$$(HF)^*G=\frac{1.5\times10^{-9}K}{z(z-1)} \tag{28}$$

Fig. 9 gives a plot of the locus of the roots of $1+(HF)^*G$ as given by equation 28. The system is now seen to be stable for all positive values of $K$ less than $6.7\times10^8$, and for $K$ less than $1.67\times10^8$, the loci remain entirely on the real axis. Note that there is a substantial increase in the allowable $K$ when the sampling frequency is increased from 1 kc to 100 kc.

For any sampling frequency greater than 100 kc the approximations that $\epsilon^{-10T}$ and $\epsilon^{-50T}$ are both unity still hold, and in fact, become even more accurate. The shape of the root loci is dentical to that shown in Fig. 9. However, the values of $K$ which correspond to points where the loci cross the unit circle vary with the sampling frequency and are found to be:

$$K=\frac{6.67\times10^3}{T} \tag{29}$$

Equation 29 shows that for high enough sampling frequencies (small enough sampling periods) the allowable feedback-loop gain $K$ increases and is directly proportional to sampling frequency (inversely proportional to sampling period). In the limit as the time between samples goes to zero, the allowable $K$ is seen to approach infinity. This result is not surprising, since the equivalent continuous reactor control system is stable for all positive $K$.

## Conclusions

According to the preceding analysis, a reactor control system consisting of a sampler, delay, clamper, and a motor-control rod-drive mechanism certainly appears to be stable. Moreover, for a sampling frequency of 1 kc the allowable feedback-loop gain seems tolerable. If the sampling frequency were increased to

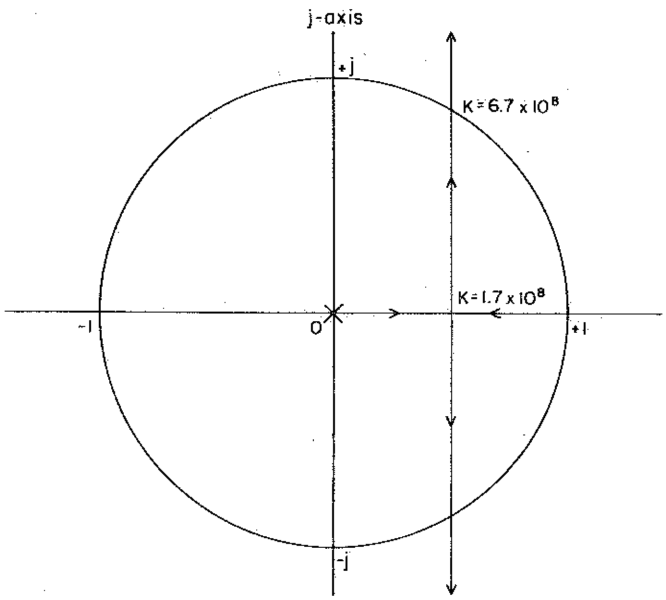


Fig. 9. Locus of roots of $1+(HF)^*G$ $T=10^{-5}$ seconds

$10^5$ kc, or greater, the allowable gain is increased substantially and, in fact, becomes directly proportional to the sampling frequency. In the limit of infinite frequency (a continuous system) the allowable gain approaches infinity. It does appear, however, that excessively high sampling frequencies are not necessary since sampling at the selected frequency of 1 kc seems more than sufficient to result in a stable system.

Accuracy studies were not made for the described computations. However, the nature of digital computation does lend itself to high accuracies. In general, it may be expected that any reactor control quantities calculated by digital methods could be computed to the same degree of accuracy as the original data.

The designed reactor control computer appears considerably more flexible than presently used analog reactor control systems. The reference levels and scale factors can be changed with considerable ease, simply by reading new numbers into the reference memory. Also, it would be possible to change the quantities to be calculated, if the nature of the substituted computations is such that a presently existing computer timing sequence could be used. The described substitution scheme adds to the flexibility afforded since it permits additional quantities to be calculated at variable time lags between individual computations. One serious limitation in the flexibility of the designed computer does exist, however. This is due to the fact that the computer is special-purpose in nature, and hence possesses nowhere near the wide degree of flexibility afforded by a general-purpose digital computer, into which almost any sequence of operations can be programed and new programs readily substituted.

The designed computer should be as reliable as any other digital computer of equal complexity. Since huge general-purpose computers much more involved than the designed computer have been built and operated successfully, a high reliability would seem possible for the designed reactor control system. In the detailed design of the system high quality components (tubes, transistors, diodes, etc.) would be used in order to minimize malfunction.

The design has not proceeded to the point where a meaningful cost estimate can be made. It may not be unreasonable to assume however, that the cost of a digital system like the one described would be higher than presently used analog systems. It would then have to be determined whether any increase in cost is justified by increases in flexibility and accuracy.

## Appendix I. Variable-Command Relations

The conditions for SCRAM are:

$n>n_m$
$(1/\tau)>(1/\tau)_m$
$S_1>S_{1m}$

The conditions for REVERSE are:

$n>n_r$
$(1/\tau)>(1/\tau)_r$
$S>S_{1r}$
$T_h>T_{hr}$
$T_c>T_{cr}$
$T_f>T_{fr}$
$(T_h-T_c)>(T_h-T_c)_r$
$p_c>p_{crh}$
$p_s>p_{srh}$
$\phi>\phi_r$
$p>p_r$
$X_s>X_r$
$L<L_r$
$F<F_r$
$p_c<p_{crl}$
$p_s<p_{srl}$
$T_{avg}<T_{avgr}$

The conditions for CUTBACK are:

$n>n_c$
$(1/\tau)>(1/\tau)_c$
$S_1>S_{1c}$
$T_h>T_{hc}$
$T_f>T_{fc}$
$T_c>T_{cc}$
$(T_h-T_c)>(T_h-T_c)_c$
$p_c>p_{cch}$
$p_s>p_{sch}$
$\phi>\phi_c$
$p>p_c$
$L<L_c$
$F<F_c$
$p_c<p_{ccl}$
$p_s<p_{scl}$
$T_{avg}<T_{avgc}$

The conditions for EXCHANGE OUT are:

$X>X_o$

The conditions for EXCHANGE IN are:

$X<X_i$

The conditions for RODS IN are:

$S_1>S_{1ref}(1+\delta)$

The conditions for RODS OUT are:

$S_1<S_{1ref}(1-\delta)$

The conditions for DECREASING FLOW RATE are:

$F>F_1$
$F>F_3$ and $(T_h-T_c)<(T_h-T_c)_2$

The conditions for INCREASING FLOW RATE are:

$F<F_4$
$F<F_2$ and $(T_h-T_c)>(T_h-T_c)_1$

## Appendix II. Timing Sequence for Calculator Operation

$P64$: sample all measurable quantities.
$P1$: read $n_k$ to sum, read $\bar{n}_{k-1}$ to addend.
$P2$: add.
$P3$: read $\bar{n}_k$ to divisor (complement), read sum to dividend.
$P4$: read $n_k$ to sum, read $n_k$ to $\bar{n}_{k-1}$ (complement), read $1/\tau$ to addend (complement digits if negative).
$P5$: add.
$P6$: read $S_1$ out of sum (complement digits if negative).
$P7$: read $T_h$ to sum, read $T_c$ to addend.
$P8$: add.
$P9$: read $T_{avg}$ out of sum.
$P10$: read $T_h$ to sum, read $T_c$ to addend (complement).
$P11$: add.
$P12$: read $(T_h-T_c)$ out of sum, read $F$ to multiplicand, read $(T_h-T_c)$ to multiplier.
Odd numbered pulses $P13$ through $P39$ are add attempts; even pulses are shift multiplier and product.
$P40$: read $P$ out of sum, read $S_{1ref}$ to $\delta S_{1ref}$.
$P41$: read $S_{1ref}$ to sum, read $\delta S_{1ref}$ to addend.
$P42$: add.
$P43$: read $S_{1ref}(1+\delta)$ out of sum.
$P44$: read $S_{1ref}$ to sum, read $\delta S_{1ref}$ to addend (complement).
$P45$: add.
$P46$: read $S_{1ref}(1-\delta)$ out of sum.

## Appendix III. Comparator Reference-Level Timing Sequence

$P1$: $n_m$
$P2$: $n_r$
$P3$: $n_c$
$P4$: $(1/\tau)_m$
$P5$: $(1/\tau)_r$
$P6$: $(1/\tau)_c$
$P7$: $S_{1m}$
$P8$: $S_{1r}$
$P9$: $S_{1c}$
$P10$: $S_{1ref}(1+\delta)$
$P11$: $S_{1ref}(1-\delta)$
$P12$: $T_{hr}$
$P13$: $T_{hc}$
$P14$: $T_{cr}$
$P15$: $T_{cc}$
$P16$: $T_{fr}$
$P17$: $T_{fk}$
$P18$: $p_{crh}$
$P19$: $p_{cch}$
$P20$: $p_{crl}$
$P21$: $p_{ccl}$
$P22$: $p_{srh}$
$P23$: $p_{sch}$
$P24$: $p_{srl}$
$P25$: $p_{scl}$
$P26$: $p_r$
$P27$: $p_c$
$P28$: $X_r$
$P29$: $(T_h-T_c)_r$
$P30$: $(T_h-T_c)_c$
$P31$: $(T_h-T_c)_1$

$P32$: $(T_J - T_c)_2$
$P33$: $\phi_r$
$P34$: $\phi_c$
$P35$: $L_r$
$P36$: $L_c$
$P37$: $X_o$
$P38$: $X_i$
$P39$: $F_r$
$P40$: $F_c$
$P41$: $F_4$
$P42$: $F_3$
$P43$: $F_2$
$P44$: $F_1$
$P45$: $T_{avgr}$
$P46$: $T_{avgc}$

## Appendix IV. Substituted Calculator Timing Sequence

$P6'$: read $S_{1ref}$ to $\delta S_{1ref}$.
$P7'$: read $S_{1ref}$ to sum, read $\delta S_{1ref}$ to addend.
$P8'$: add.
$P9'$: read $S_{1ref}(1+\delta)$ out of sum.
$P10'$: read $S_{1ref}$ to sum, read $\delta S_{1ref}$ to addend (complement).
$P11'$: add.
$P12'$: read $S_{1ref}(1-\delta)$ out of sum, read $F$ to multiplicand, read $(T_h - T_c)$ to multiplier.

## References


1. Control of Nuclear Reactors and Power Plants (book), **M. A. Schultz.** McGraw-Hill Book Company, Inc., New York, N. Y., 1955.

2. Nuclear Engineering (book), **C. F. Bonilla.** McGraw-Hill Book Company, Inc., 1957.

3. Principles of Nuclear Reactor Engineering (book), **S. Glasstone.** D. Van Nostrand Company, Inc., Princeton, N. J., 1955.

4. Progress in Nuclear Energy, Series II Reactors (book), **R. A. Charpie, D. J. Hughes, D. J. Littler, M. Trocheris.** McGraw-Hill Book Company, Inc., 1956.

5. Arithmetic Operations in Digital Computers (book), **R. K. Richards.** D. Van Nostrand Company, Inc., 1958.





November 1960 issue