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CAN AN OMNIPOTENT MAXWELLIAN DEMON BEAT THE LAWS OF THERMODYNAMICS?

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ABSTRACT

Over the past twelve decades, close to 300 papers have been written to discuss Maxwell's demon. Usually, the demon is exorcised by arguments related to irreversible processes, information theory, or computers and erasure of information. In all these discussions, the demon himself experiences some energetic and entropic cost in attempting to execute his demonic task.

Short of changing the laws of thermodynamics, we show that even an omnipotent demon, who can do whatever he pleases at no cost, would not be able to beat the laws of thermodynamics. The proof is given both in terms of engineering thermodynamics, and in terms of quantum physics.

INTRODUCTION

The purpose of this paper is to provide a rigorous exorcism (!) of Maxwell's demon based on the two premises: (1) the thermodynamic idea that entropy is a property of matter and not of the mind; and (2) the quantum-theoretic idea that in equilibrium nothing moves.

A very informative collection of articles and commentary was edited recently by Leff and Rex [1] under the title *Maxwell's Demon, Entropy, Information, Computing*. The authors review briefly about 300 references that appeared in the scientific literature over a period of twelve decades, and reprint fully about 30 of the seminal publications, all pertaining to the demon.

As we will see later, Maxwell's demon is an attempt to get around the limitations imposed by the laws of thermodynamics, such as to violate the principle of nondecrease of entropy or, equivalently, to make possible the manufacture of a perpetual motion machine of the second kind. Even though no one claims to have succeeded in this goal, it seems to me that hope springs eternal, and that the arguments used are not fully consistent with the fundamental aspects of thermodynamics and quantum theory.

The paper is organized as follows. Maxwell's definition of the demon, and a brief history of the demon's more than 120 year old life are given in the second section, an engineering thermodynamics discussion by means of an energy versus entropy graph in the third section, a quantum-theoretic analysis in the fourth section, and conclusions in the last section.

BRIEF HISTORY

What is Maxwell's demon?

The Random House Dictionary of the English Language (Second Edition, 1988) defines the demon as follows.

> Maxwell's demon: A hypothetical agent or device of arbitrarily small mass that is considered to admit or block selectively the passage of individual molecules from one compartment to another according to their speed, constituting a violation of the second law of thermodynamics.

This definition is so loosely stated as to include innumerable processes that do occur every day and do not violate the laws of thermodynamics. For example, in a system with two counterflow beams of particles, the separation of the two beams occurs naturally. It involves neither interference by any hypothetical agent, nor violation of any laws of physics.

Besides, the dictionary definition does not fully represent what Maxwell had in mind when he introduced his demon in his 1871 book, *Theory of Heat*. Maxwell wrote [2]:

Before I conclude, I wish to direct attention to an aspect of the molecular theory which deserves consideration.

One of the best established facts in thermodynamics is that it is impossible in a system enclosed in an envelope which permits neither change of volume nor passage of heat, and in which both the temperature and the pressure are everywhere the same, to produce any inequality of temperature or of pressure without the expenditure of work. This is the second law of thermodynamics, and it is undoubtedly true as long as we can deal with bodies only in mass, and have no power of perceiving or handling the separate molecules of which they are made up. But if we conceive a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are still as essentially finite as our own, would be able to do what is at present impossible to us. For we have seen that the molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower ones to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics.

This is only one of the instances in which conclusions which we have drawn from our experience of bodies consisting of an immense number of molecules may be found not to be applicable to the more delicate observations and experiments which we may suppose made by one who can perceive and handle the individual molecules which we deal with only in large masses.

In dealing with masses of matter, while we do not perceive the individual molecules, we are compelled to adopt what I have described as the statistical method of calculation, and to abandon the strict dynamical method, in which we follow every motion by the calculus.

It is noteworthy that Maxwell's system is initially in what we call today a thermodynamic equilibrium or stable equilibrium state. It is only certain changes of such states that, if brought about by the demon, would constitute violations of the laws of thermodynamics.

Maxwell's sharp-witted being was subsequently nicknamed "Maxwell's intelligent demon" by Thomson [3]. He wrote "The definition of a demon, according to the use of this word by Maxwell, is an intelligent being endowed with free will and fine enough tactile and perceptive organization to give him the faculty of observing and influencing individual molecules of matter."

Maxwell's and Thomson's references to intelligent beings left the strong impression that such beings are not subject to the same laws of physics as inanimate matter, an impression that is contrary to contemporary scientific and engineering evidence.

Much later, Smoluchowski [4] tried to alleviate this impression by arguing that the demon could be conceived as a simple automatic apparatus, such as a trap door, and that the apparatus could not achieve the sorting of the molecules because of its own Brownian motion. But then he added [5]: "As far as we know today, there is no automatic, permanently effective perpetual motion machine, in spite of molecular fluctuations, but such a device might, perhaps, function regularly if it were appropriately operated by intelligent beings."

There are two interesting notions in the last quotation. One is the reintroduction of the role of intelligence, shifted from the apparatus to the operator, and the other the use of the term perpetual motion machine. I will address the second notion in the next section. Regarding the shift from apparatus to operator, it seems to me that the change is a distinction without a difference.

The apparent ability of intelligent beings to violate the laws of thermodynamics was addressed by Szilard in his famous paper [6] "On the Decrease of Entropy in a Thermodynamic System by the Intervention of Intelligent Beings." He argued that the act of measurement, by which the demon determines the velocity of the molecule, is necessarily accompanied by an entropy increase sufficient to compensate or overcompensate for the entropy decrease that accompanies the separation of the molecules into two types, fast and slow. He was vague about where the entropy increase occurs. Later on, it was assumed that the increase appears in the entropy of the universe.

Szilard's ideas led to the concept of bit of information, and created a fertile ground for the development of information theory, cybernetics, and computing.

Brillouin [7] and Gabor [8] followed up on the thermodynamic cost of measurement by using the quantum nature of light, and appeared to have resolved Maxwell's demon's puzzle. Brillouin wrote: "Is it actually possible for the demon to see the individual atoms?...The demon is in an enclosure at equilibrium at constant temperature,..., and it is impossible to see anything in the interior of a black body...The demon would see thermal radiation and its fluctuations, but he would never see the molecules."

Next, Brillouin considers a dissipative measurement procedure in which the demon observes the molecules by using photons that are not in equilibrium with the air. He concludes that the demon must use photons more energetic than the photons comprising the thermal background and, therefore, dissipate energy.

Later on, Landauer [9] introduced the concept of logical irreversibility in connection with information-discarding processes in computers. The work of Landauer was extended by Bennett [10], who argued that a computing automaton can be made logically reversible. However, Bennett's extension required a reservoir.

It is noteworthy that all refutations of the demon in references 6 to 10 rely on entropy, but entropy that refers to both the system and the demon, and also to some irreversibility.

In my judgment all these arguments, plus many more that are discussed in much greater detail in reference [1], are perhaps correct but not germane to the issue at hand, namely, whether there exist processes which violate the laws of thermodynamics. The reason for this judgment is the well-known and powerful fact that thermodynamics specifies limits on the performance of a system in terms of its initial and final states, and the flows at its boundary, that is, with reference to a process, and not with reference to the specific equipment and procedures that may be used to achieve the process. For example, the so-called available energy of 1 kg of methane (largest work that can be done by the methane plus stoichiometric air in combination with the environment) is determined solely by the methane and the environment, and not by the steam engine, fuel cell, gas turbine, or operator that may be used for that purpose. Another example is a work-producing heat engine operating between two fixed temperatures T_1 and T_2. Because T_1 is fixed, the engine receives energy Q_1 and, concurrently, entropy Q_1/T_1. Since it produces only work (transfers only energy through the shaft), the engine must dispose of at least the entropy Q_1/T_1 at temperature T_2. But such disposition is accompanied unavoidably by concurrent energy transfer $T_2(Q_1/T_1)$. So the energy that remains for transfer across the shaft is $Q_1 - T_2(Q_1/T_1)$. To be sure, the engine may have all sorts of deficiencies of its own, and may generate entropy that must be discarded at temperature T_2 together with Q_1/T_1. Then the energy discarded at T_2 is greater than $T_2(Q_1/T_1)$ and, therefore, what remains for the shaft is less than $Q_1 - T_2(Q_1/T_1)$.

In support of the judgment, I offer two arguments, one based on engineering thermodynamics, and the other on quantum-theoretic considerations. The engineering argument is discussed in terms of an energy versus entropy graph, and proves that a perpetual motion machine is impossible because it violates the principle of nondecrease of entropy. The quantum-theoretic argument is discussed in terms of the canonical distribution, and proves that no molecules can be sorted out as fast and slow because, in a stable equilibrium (thermodynamic equilibrium) state, no molecule moves!

ENGINEERING THERMODYNAMICS CONSIDERATIONS

In view of the current understanding of engineering thermodynamics, a complete definition of Maxwell's demon is the same as that of a perpetual motion machine of the second kind.

A perpetual motion machine of the second kind (PMM2) is any system B undergoing a cyclic process that produces no external effects except the rise of a weight in a gravity field, and the change of another system A from an initial stable equilibrium (thermodynamic equilibrium) state A_0 to a

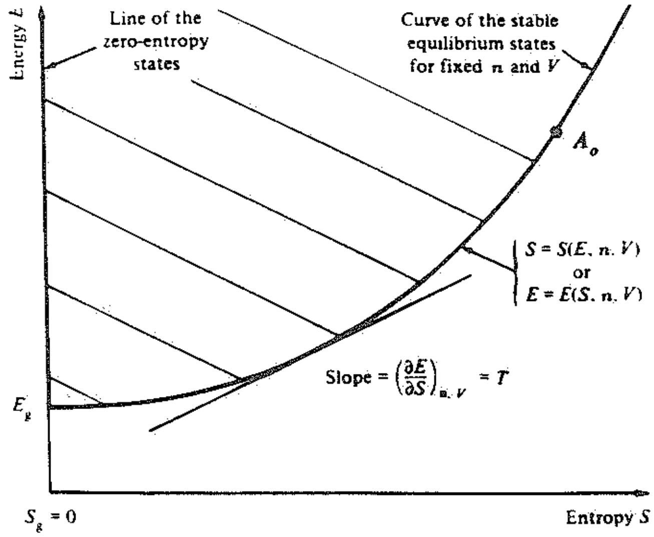

Figure 1

different final state A_1 corresponding to the same values of amounts of constituents and volume (external parameters) as state A_0. State A_1 may have the same energy as A_0, but then the rise of the weight is null (principle of energy conservation).

In this definition, system A is Maxwell's air, and system B is Maxwell's demon. Moreover, the definition excludes both contributions of the demon to any of the balances that must be satisfied, such as the energy and entropy balances, and the exchange of constituents with the environment. The former exclusion is valid because system B (the demon) experiences a cycle, and the latter exclusion because neither the amounts of constituents of A nor those of the weight change.

The feasibility of a PMM2 can be investigated by using the energy versus entropy graph of system A, a graph that provides a pictorial representation of many results of thermodynamics. As discussed in reference 11 (see also G. P. Beretta and E. P. Gyftopoulos, "A Novel Sequence of Exposition of Engineering Thermodynamics," Proceedings of this annual meeting), the projection of the state-space of system A (the air in Maxwell's language) with given values of amounts of constituents and volume on the energy versus entropy plane has the shape of the cross-hatched area shown in Figure 1. In words, all states that share the given values have property values that project on the area between the line of zero entropy and the curve of the stable equilibrium states. The initial state A_0 is on the latter curve.

If a PMM2 (a demon) were feasible, it would extract only energy from A beginning with state A_0 and ending with a state A_1 of smaller energy and the same or larger entropy (principle of nondecrease of entropy) than the energy and entropy of A_0. But under the specified conditions, all states of energy smaller than or equal to that of A_0 have also smaller entropy. It follows that a PMM2 or Maxwell's demon is impossible because there exists no final state of system A which is compatible with the demon's process.

So, even if the demon were omnipotent and could measure and see molecules or do whatever he pleased at no cost to him whatsoever, he would not be able to accomplish his task, not because of his own limitations but because of limitations imposed by the characteristics of the system on which he is working. Altering the latter limitations requires a change of the laws of physics, laws which are independent of the demon's tactile and perceptive abilities.

Of course, if the initial state of A is not stable equilibrium and, therefore, lies somewhere within the cross-hatched area but not on the stable-equilibrium state curve of the graph in Figure 1, even a less competent demon, let alone a smart one, could extract only energy from A without violating the laws of thermodynamics. For, we see from the graph that then many states exist with lower or equal energy and equal or larger entropy than the energy and entropy of the initial state. Accordingly, the demon can extract only energy from the system in a manner entirely consistent with the predicaments of the principle of nondecrease of entropy.

A trivial illustration of the last two conclusions are two identical, electricity storage batteries, both having the same energy but one having been internally discharged and the other having just been charged. Clearly, a little boy or girl, knowing only how to press a small button (switch), gets no electricity from the discharged battery (stable equilibrium state), and lots of electricity from the charged battery (not a stable equilibrium state) without any cost to him or her, and without any difficulty.

To some the argument given here may appear circular because I use the laws of thermodynamics to prove that they cannot be violated. However, I believe that the argument is not circular for the following reasons. If entropy is not a property of matter, then all the energy of the air above its ground-state energy is available for transfer to other systems. An elegant quantum-mechanical proof of this assertion has been provided by von Neumann, who proves that any wave function can be connected to any other wave function by an adiabatic (work only) process. Given the initial and final wave functions and a time interval, we can calculate the precise interaction Hamiltonian operator that accomplishes this process. So we know exactly what Maxwell's demon must do to carry out his task and, therefore, there are no limitations.

A possible counterargument might be that the demon does not know the initial state of the system. This counterargument is contrary to innumerable experiences. As the example with the two batteries illustrates, even an illiterate child knowing nothing about batteries can get energy out of a charged battery (nonequilibrium state), whereas no one has ever been able to get energy out of an identical discharged battery having the same energy as the charged battery. This sharp difference is not related to the user's knowledge of the initial states of the batteries. There must be a fundamental difference that goes beyond one's knowledge of initial conditions. This difference is captured by the introduction of entropy as a property of matter.

If entropy is a property of matter, then Maxwell's demon must cope with the rules of behavior of this property, one of which is that, in adiabatic processes, entropy cannot decrease. Because entropy emerges as a result of the laws of thermodynamics, we say that the demon cannot violate them. This limitation arises not from the demon's shortcomings but from the properties of the system on which he is working.

It is noteworthy that, in the scientific literature, each refutation of Maxwell's demon is based on some kind of entropic argument, but entropic argument that relates to the demon and not the system on which he operates. My observation is that the entropic limitation imposed by the system is sufficient to exorcise the demon without any additional help that may be provided by his clumsiness.

QUANTUM-THEORETIC CONSIDERATIONS

In quantum thermodynamics, stable equilibrium states are represented by energy eigenstates, and canonical or grand-canonical quantum probabilities (not statistical probabilities). The recognition of these facts provides another exorcism of Maxwell's demon. The reason is as follows.

For simplicity of the argument, I will consider a system A having a value for the amount of each type of constituent equal to an eigenvalue of the corresponding number operator, and a fixed volume. The most general description of the stable equilibrium (thermodynamic equilibrium) states of A is quantum-mechanical not only because classical mechanics is a limiting case of quantum physics but, more importantly, because there exist quantal aspects which can be neither represented classically nor overlooked. Specifically, the energy E, entropy S, and quantum-mechanical probabilities x_i are given by the relations

$$E = \sum_i x_i \epsilon_i \tag{1}$$

$$S = -k \sum_i x_i \ln x_i \tag{2}$$

$$x_i = \exp(-\beta \epsilon_i) \Big/ \sum_i \exp(-\beta \epsilon_i) \tag{3}$$

where ϵ_i is the ith energy eigenvalue, k is the Boltzmann constant, a different i is used even for degenerate eigenstates, and the coefficient β is determined solely by the value of E.

As it is very well known, the distribution x_i, the so-called canonical distribution, is found by maximizing S subject to a given value E and $\sum_i x_i = 1$. Of course, the resulting x_i's are the eigenvalues of the density operator in the energy representation.

Prior to discussing the implications of these results about Maxwell's demon, it is noteworthy that, for fixed ϵ_i's (fixed volume), one can evaluate the differentials $(dE)_\epsilon$ of E, and $(dS)_\epsilon$ of S, and find the ratio $(\partial E/\partial S)_\epsilon$, where subscript ϵ

stands for all eigenvalues ϵ_i being kept constant. Using equations (1) to (3), the result is

$$\left(\frac{\partial E}{\partial S}\right)_{\epsilon} = \frac{1}{k\beta} \tag{4}$$

So, because β is determined solely by the value of E, the value of $(\partial E/\partial S)_{\epsilon}$ also is determined solely by the value of E. But in classical thermodynamics, $(\partial E/\partial S)_{\epsilon}$ = T = temperature. Accordingly, $T = 1/k\beta$ and, therefore, T is fixed solely by β, that is, solely by E. This result shows beyond doubt that the value of T is determined solely by E and not by either the temperature of a reservoir, or weak interactions between member-systems of an ensemble as sometimes claimed in statistical interpretations of thermodynamics.

The energy eigenvalues and eigenfunctions are found by solving the energy eigenvalue problem. A very well known result of quantum physics is that each energy eigenstate is an equilibrium or stationary state — a state that does not change with time — and that the value of the velocity of the molecules in each energy eigenstate equals zero. It follows that the combination of energy eigenstates represented by the canonical distribution is also an equilibrium state and that the value of the velocity of the molecules represented by this distribution equals zero. In the conclusions just cited, the value of the velocity refers to the entire system. In dilute systems, such as Maxwell's air, we can express the velocity associated with each system energy eigenstate in terms of velocities of individual molecules. We do so by using the one-particle approximation. In this approximation, each energy eigenstate of the entire system is represented by a symmetric or antisymmetric combination of energy eigenstates of a one molecule system — symmetric for bosons and antisymmetric for fermions. Because the value of the velocity of each one-particle energy eigenstate is zero, the values of the velocities of the molecules of the system distributed over these eigenstates is also zero. The zero value of the velocity either of all the molecules collectively in an energy eigenstate, or of individual molecules allocated to one-particle energy eigenstates, is a quantum-theoretic result that has no classical analogy. It represents one of the great triumphs of quantum physics. It has helped us understand many phenomena, including the stability of atoms and molecules.

The significance of these observations is that Maxwell's demon (doors, traps, and the like) cannot do his job because in thermodynamic equilibrium there are no moving molecules to be sorted out. So to speak, if the demon is seated at Smoluchowski's door, nobody will ever go by in one direction or the other because all are at a standstill (energy eigenstate). The poor fellow would be as sad as a "Maytag" washing-machine repairman because, even after many costly measurements, he would conclude that the molecules, both collectively and individually, are idle — immobile.

CONCLUSIONS

It is shown for the first time that Maxwell's demon is not possible either from the engineering-thermodynamic point of view, or from that of quantum physics. From the former point of view, he is limited by the states that a system can attain starting from a stable equilibrium state. From the latter point of view, he is limited by the immobility of the molecules under scrutiny.

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